Teacher's Guide

# Green Globs and Graphing Equations

Apple

SUNBURST
COMMUNICATIONS

# PERMISSIONS

All SUNBURST material is copyrighted. However, SUNBURST does give the purchaser the following permission:

1. You have permission to reproduce any student worksheets in this guide for your classroom use. You should not, however, copy the whole guide.

2. You have permission to use Lab Packs within one site. You should not, however, divide the package and use the diskettes in more than one building.

Authors: Sharon Dugdale
David Kibbey

Teacher's Guide: Lois Edwards
Lisa Paul

This guide is based on materials copyrighted (c) 1983, 1985 by Sharon Dugdale and David Kibbey.

Copyright (c) 1986
Sunburst Communications, Inc.
Pleasantville, NY 10570

Apple is a registered trademark of Apple Computer, Inc. of Cupertino, CA

For information or a free Microcomputer Courseware Catalog, write:
SUNBURST COMMUNICATIONS, INC.
39 WASHINGTON AVE.
PLEASANTVILLE, NEW YORK 10570

Call toll-free: (800) 431-1934
(In New York, Alaska, and Canada, call collect (914) 769-5030.)

GREEN GLOBS AND GRAPHING EQUATIONS

Table of Contents


Introduction to the Teacher..... 1
Teacher Instructions..... 4

EQUATION PLOTTER..... 6
Program Description..... 7
Ideas for Classroom Use..... 8
Student Materials..... 9

LINEAR AND QUADRATIC GRAPHS.....40
Program Description.....41
Ideas for Classroom Use.....42
Student Materials.....45

GREEN GLOBS.....47
Program Description.....48
Ideas for Classroom Use.....51
Student Materials.....55

TRACKER.....57
Program Description.....58
Ideas for Classroom Use.....60
Student Materials.....62

Apple: Working with Your Computer.....65

"What Happens If...?" -- Sunburst Courseware and Warranty.....66

## INTRODUCTION TO THE TEACHER

Green Globs and Graphing Equations provides mathematical environments for students to manipulate and explore. This may be quite different from the mathematics activities students have experienced in the past. Students who use Green Globs become creative learners, developing new strategies and innovative techniques.

The mathematical content of this package focuses on the relationship between equations and their graphs. The programs are appropriate for a wide range of student abilities and experience levels. Beginning algebra students or pre-algebra students who are familiar only with graphs of straight lines can benefit from these programs. Math analysis students who use trigonometric and discontinuous algebraic functions will also be challenged.

### Brief Program Descriptions

EQUATION PLOTTER is a utility program that plots the graphs of entered equations.

LINEAR AND QUADRATIC GRAPHS displays a graph of a certain type (line, parabola, circle, ellipse, hyperbola). The student enters an equation that matches the graph.

GREEN GLOBS is a game in which thirteen "green globs" are randomly placed on the grid. Students earn points by entering equations that pass through as many green globs as possible.

TRACKER is a game in which graphs are hidden on a grid. The student gathers clues to determine the equations of the hidden graphs by using "probe shots" (horizontal and vertical lines) to find the location of the graphs.

### Suggested Uses

EQUATION PLOTTER is effective as a demonstration tool in classroom presentations to introduce graphs of equations and to illustrate the concept of "what if" (For example, What if we changed the equation? How would the graph change?) It is also valuable as a student activity with worksheets such as those included in this guide.

LINEAR AND QUADRATIC GRAPHS is an individual or small group activity that will help students master methods of transforming graphs by altering basic equations.

GREEN GLOBS is an individual or small group activity that enables students to apply concepts relating equations and their graphs in a motivating, problem-solving situation. It helps students develop creative strategies using equations.

TRACKER is an individual or small group activity that reinforces the understanding of linear and quadratic relations in a game setting.

## Order of Use

The programs can be used in any order; however, LINEAR AND QUADRATIC GRAPHS is recommended as preparation for TRACKER. The hidden graphs in TRACKER are the same as the graphs in LINEAR AND QUADRATIC GRAPHS.

The worksheet activities in the EQUATION PLOTTER section provide practice that students may find useful in GREEN GLOBS.

## Time Requirements

The amount of time needed for effective use of these programs varies greatly. A few minutes is sufficient for a demonstration using EQUATION PLOTTER; a thirty-minute session is enough to use one part of LINEAR AND QUADRATIC GRAPHS. Several hour-sessions are necessary to develop a range of strategies for playing GREEN GLOBS.

Teachers may find the time commitment of these activities greater than the time they typically devote to class presentations of this content. However, it should be noted that students here are actually applying mathematics and developing creative strategies, not seeing or listening to a demonstration.

## The Teacher's Role

Teachers who use Green Globs and Graphing Equations effectively act as facilitators. By planting a few carefully-placed comments and asking questions, they encourage students to explore their own ideas.

Teachers should encourage students to learn from each other, to build on others' ideas, and to explain their ideas to their group. Teachers can spark continued interest and creativity in students by subtly guiding them toward new strategies that suit their ability level and experience.

It is not an easy task to be aware of the students' current strategies, share their enthusiasm about their latest discovery, allow them to bask in this achievement by applying it several times and extending it to new cases, and yet at the same time encourage them to move on to new ideas.

## The Student's Role

Students should try to develop basic strategies that can be applied to many graphs rather than to memorize formulas. They can do this by experimenting, by using trial and error, and by learning from their "mistakes." Students should be encouraged to be creative and innovative, to share ideas, and to build on others' ideas.

Students may want to work individually or in groups of two or three.

TEACHER INSTRUCTIONS

## Instructions for Use

Option 0 on the diskette, "How to Use These Programs," explains the method of entering information and the use of special keys in these programs:

When a program writes an "➤" on the screen, it is waiting for you to make a choice or answer a question. You type your response, and then press the RETURN key to go on.

If you make a mistake while typing your response, use the "←" key to erase. The programs call it the ERASE key.

You can make changes in the middle of an equation: use the "<" and ">" keys to move the pointer to the place you want to change. The pointer consists of an underline and a vertical bar. You can erase above the underline or insert at the vertical bar.

Option 0, "How to Use These Programs," provides a place to practice writing and changing equations.

Generally, you press the RETURN key to go on, and the ESC key to back up.

When you want to leave a program, press the ESC key several times and you will back up to the index of programs.

Each program also contains instructions that students should read before using the program.

## Correcting Monitor Distortion

The monitor may need to be adjusted to eliminate distortion in the graphs.

Select the "How to Use These Programs" option on the diskette. Check to see if the circle shown in the test appears flattened or otherwise distorted.

If the test circle is distorted, adjust the vertical size and linearity controls on the back of the monitor. It may be necessary to have a computer repair person make these adjustments on some monitors.

## Teacher Change Option

Two of the programs, GREEN GLOBS and TRACKER, keep lists of the best scores and the names of the students who earned them.

Teachers can change these names or delete the records by using the "Management Options." To access this option, press Control-T on the initial screen, the main menu screen, or any Record Scores display.

### Password

In the management option, a password system inhibits students from deleting records or changing names. Be sure to keep your password confidential. Anyone who knows the password can change the records and change the password.

Choose a password longer than two letters. Choose one that is not likely to be guessed by others.

If you want to alter student records without deleting them all, but you forget your password, you'll need to send the disk to us. We'll return your disk with no password on it, and with your existing student records intact. You can then select a new password. If you forget the password you've chosen, and you want to delete all student records, call our toll-free number (800-431-1934); we will send you a new disk, and you can select a new password.

### Using the Management Options

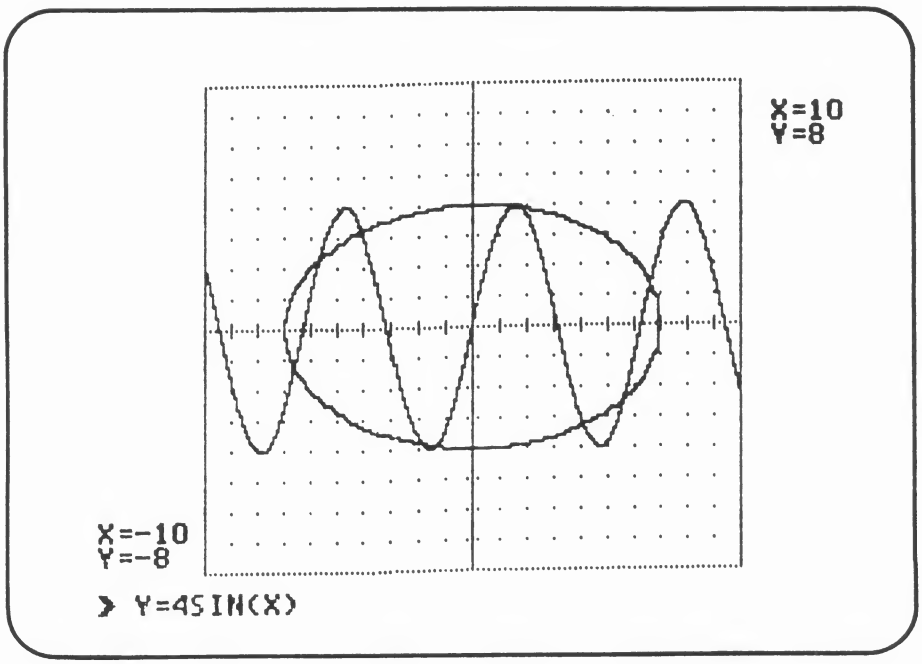
Press Control-T on the initial title screen, the main menu, or any Record Scores display.

Enter your password, as requested.

Follow the instructions on the diskette to delete records or change names. You can delete records or change names in:

1. GREEN GLOBS Novice Game
2. GREEN GLOBS Expert Game
3. TRACKER Novice Game
4. TRACKER Expert Game

EQUATION PLOTTER




Grade Level: 6 - adult

Prerequisite Skills: use of a coordinate grid, graphing equations

Type: graphing utility for student or teacher use

Description

EQUATION PLOTTER plots the graphs of equations entered at the keyboard. The grid can be either rectangular (as used in the other programs in this package) or square. The grid bounds can be specified. Acceptable forms of equations include conics, as well as equations where:

y equals a function of x, or
x equals a function of y.

Functions may include trigonometric, square root, absolute value, natural logarithm, and exponential functions.

Objectives

1. to recognize basic shapes of graphs of equations
2. to discover relationships between equations and their graphs
3. to manipulate graphs by altering the corresponding equation

(Note: Objectives may vary depending on the instructional activities.)

PROGRAM DESCRIPTION

## The Coordinate Grid

The equations are graphed on either a rectangular or a square grid. When the program is started, the bounds for the rectangular grid are X = -10 to +10 and y = -8 to +8. The bounds for x and y can be changed. They can be up to six characters long and can include $\pi$ (pi) (for example, [.003, .004] and [-3 $\pi$ /2, 2 $\pi$ ] are valid).

## Conic Sections

Lines, parabolas, circles, ellipses, and hyperbolas are accepted if their equations contain not more than one x and one y. (Equations like $y = x^2 + 3x + 5$ which contain more than one x term are recognized as General Functions, described below.)

The following are examples of equations that are recognized as conics:

| | |
|---|---|
| 4x - .5y = 7 | $(x-3)^2/16 - (2y+3)^2/(2.4^2+3)$ |
| y + 8 = 12/(2x-3) | 4(x+1) - .5(2y-2/3) + 7 = 3 |
| (2x-3)(y+8) = 12 | $y - 3 = 2(2x+4)^2/5 + 1$ |

The program also recognizes and plots degenerate conics, such as these:

| | |
|---|---|
| (x+4)(y+5) = 0 | $(x+1)^2 + 3(y-2)^2 = 0$ |
| $(y-2)^2 = 0$ | $3(x-2)^2/4 - (y+1)^2/3^2 = 0$ |

Conics will graph accurately, including ones that are very steep, like these:

| | | |
|---|---|---|
| $y = 10000(x-2)^2$ | (x+5)(y-1000) = 1 | y = .0001/x |

## General Functions

Equations which are not recognized as conics can be graphed if they are written in the form

y = a function of x or x = a function of y

Equations may include the trigonometric functions: SIN, COS, TAN, CSC, SEC, COT, and ATN (arctangent). Also accepted are SQR (square root), and ABS (absolute value), LOG (natural logarithm), and EXP (exponential function). The following are examples of equations that are recognized as general functions:

| | |
|---|---|
| y = 2x + EXP(x) | $y = x^5 - .6x^2 + x^3/(x+1)^2$ |
| $y = x^2 SIN(2-x)$ | $x = LOG(TAN^2(y))$ |
| y = SQR(x) -1/x | $x = ABS(y^2-y) + 1$ |

General functions are evaluated at increments of one screen dot, starting at the lower bound of the independent variable. Graphs of general functions that are very steep or have very short periods may not plot with the desired accuracy. Accuracy can be improved by choosing the bounds of the independent variable to cover a smaller interval.

See Worksheet #1--Instructions on page 9 for more details on the program.

IDEAS FOR CLASSROOM USE

Teacher Demonstration Tool

As a classroom demonstration tool for teachers, EQUATION PLOTTER quickly and accurately plots graphs of equations. It can be used on a large screen monitor or video projector to enhance presentations involving graphs of equations.

The teacher or students can choose equations to be graphed and see the result on the screen. The general "what if" question can be asked and the outcome quickly observed. For example, what if we added 4 to the equation? what if we divided the equation by 2?

The teacher may want to enter an equation and show only the graph to the students. Then the class might discuss ways to determine an equation that matches the graph. This activity is recommended as preparation for the LINEAR AND QUADRATIC GRAPHS program.

Student Utility

Students can use this graphing utility to explore the relationships between equations and graphs. Using worksheets such as those included in this manual, students can learn mathematical concepts and experiment with graphing techniques.

Worksheets for Students

Worksheet 1 - Instructions
This sheet provides students with the information they need to use this program.
Worksheet 2 - Graphing Lines
Worksheet 3 - Graphing Parabolas
Worksheet 4 - Graphing Circles
Worksheet 5 - Graphing Ellipses
Worksheet 6 - Graphing Hyperbolas
These introductory activities may be useful as preparation for LINEAR AND QUADRATIC GRAPHS and TRACKER. They also provide computer graphing activities to accompany the corresponding chapters in a textbook.
Worksheet 7 - Exploring Roots
Worksheet 8 - Passing Through Points
These activities can help prepare students to use the GREEN GLOBS game.
Worksheet 9 - Advanced Techniques -- Sums of Functions
Worksheet 10- Advanced Techniques -- Trigonometric Functions
Worksheet 11- Advanced Techniques -- Sums with Trigonometric Functions
Worksheet 12- Advanced Techniques -- Absolute Value Functions
Worksheets 9-12 are intended for students who have had considerable experience with GREEN GLOBS and are ready to investigate advanced mathematical concepts.

EQUATION PLOTTER
#1 - INSTRUCTIONS

EQUATION PLOTTER plots graphs of equations on a coordinate grid. You enter the equations from the keyboard.

Changing Bounds on the Grid

You select either a rectangular or square grid. You can change the bounds anytime by pressing CTRL-S. When you start, the rectangular grid is bounded by x = -10 to x = +10 and y = -8 to y = +8.

The bounds can be expressed in terms of π (pi) by using the letter P. You may need to change the bounds in order to see your graph more accurately or to examine its behavior over a larger or smaller range.

Entering Equations

The equations you enter can include the following:

- lines, parabolas, circles, ellipses, hyperbolas (for these, write your equation using x and y only once)

- equations where y equals a function of x

- equations where x equals a function of y

- trigonometric functions, SIN, COS, TAN, CSC, SEC, COT, ATN (arctangent)
  square roots, SQR
  absolute value, ABS
  natural logarithms, LOG
  exponential functions, EXP

Clearing the Grid

You can plot several graphs on one grid. To clear the grid and graph another equation on a blank grid, press ESC.

Changing the Equation

Your equation remains on the screen after its graph has been plotted. If you want to change the equation you can press the back arrow key ← to erase one character at a time. You can erase part of the equation and make changes before graphing again, or you can use the " < " and " > " keys to make changes in the middle of your equation.

If you want to erase the entire equation and start another, just press RETURN.

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#2 - GRAPHING LINES

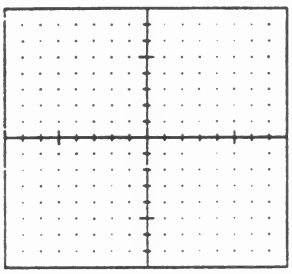
Enter each set of equations on the same coordinate grid. Observe the differences and similarities among the graphs.

Use ESC then RETURN to clear the grid. Use the back arrow key ← to save typing.

Sketch the graphs on the grids at the right.

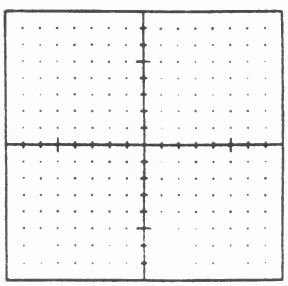
1. x = 5
   x = -6
   x = 3.5

   How might you describe all of these lines? _____



2. y = 2
   y = -4
   y = -5.5

   How might you describe all of these lines? _____

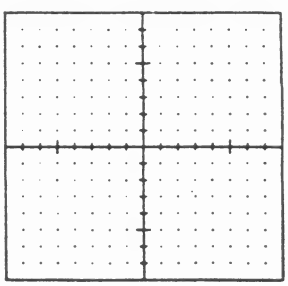


3. y = x
   y = x + 3
   y = x + 6
   y = x - 4
   y = x - 1

   a. What is alike about all these lines? _____

   b. Where does each line cross the y axis? _____

   c. What happens to the graph when a constant is added to y = x? _____
   _____

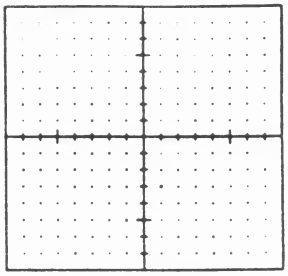


SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#2 - GRAPHING LINES - page 2

4. $y = x$
$y = 2x$
$y = 5x$

$y = -x$
$y = -3x$
$y = -6x$



a. What happens to the graph when x is multiplied by a positive number greater than 1? _____

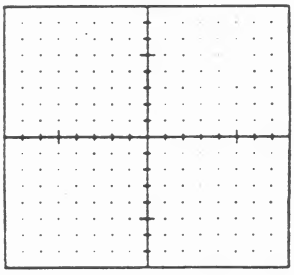
b. What happens to the graph when x is multiplied by a negative number less than -1?_____

5. $y = \frac{1}{2}x$

$y = \frac{1}{4}x$

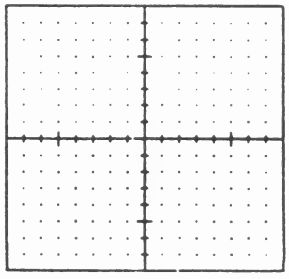
$y = -\frac{1}{3}x$

$y = -\frac{1}{5}x$



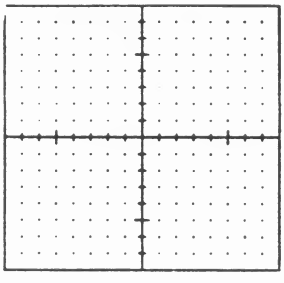
a. What happens to the graph when x is multiplied by a number between 0 and 1?_____

b. What happens to the graph when x is multiplied by a number between -1 and 0? _____



6. $y = 2x$
$y = 2x + 3$
$y = 2x - 7$
$y = -2x$
$y = -2x + 5$

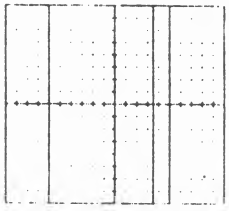
7. $y = .5x$
$y = .5x + 2$
$y = .5x - 4$
$y = -.5x$
$y = -.5x + 5$



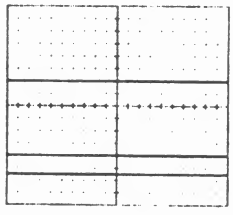
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #2 - GRAPHING LINES

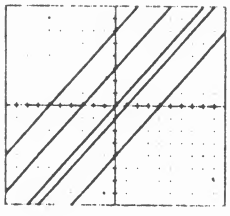
1. all are vertical



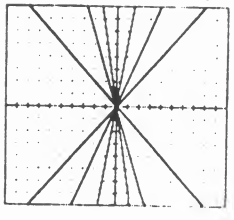
2. all are horizontal



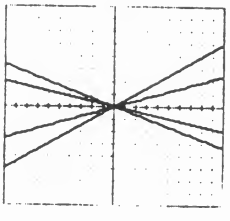
3a. all have same slope
(same tilt, same direction)
b. 0, 3, 6, -4, -1
c. it moves up or down on the grid



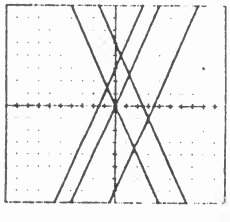
4a. it gets steeper, closer to vertical
b. it gets steeper, closer to vertical but it slopes in the opposite direction



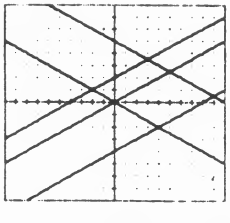
5a. it gets flatter, closer to horizontal
b. it gets flatter



6.



7.



SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#3 - GRAPHING PARABOLAS

Enter each set of equations on the same coordinate grid using the program EQUATION PLOTTER. Observe the differences and similarities among the graphs.

Use ESC then RETURN to clear the grid. Use the back arrow keys ← to save typing.

1.

| | vertex |
|---|---|
| $y = x^2$ | _____ |
| $y = (x+1)^2$ | _____ |
| $y = (x+3)^2$ | _____ |
| $y = (x-2)^2$ | _____ |
| $y = (x-5)^2$ | _____ |

What happens to the graph when you add a constant to the x term before it is squared? _____

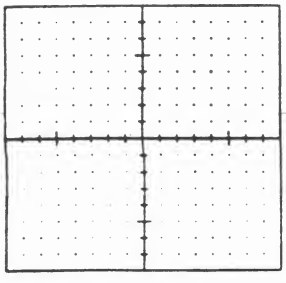
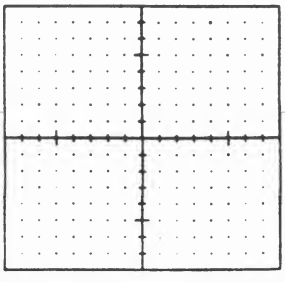
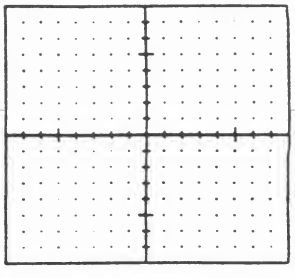
2.

| | vertex | Does it touch or cross the x-axis? List the points | |
|---|---|---|---|
| $y = x^2$ | _____ | _____ | _____ |
| $(y+2) = x^2$ or $y = x^2-2$ | _____ | _____ | _____ |
| $(y+4) = x^2$ or $y = x^2-4$ | _____ | _____ | _____ |
| $(y-3) = x^2$ or $y = x^2+3$ | _____ | _____ | _____ |
| $(y-6) = x^2$ or $y = x^2+6$ | _____ | _____ | _____ |

What happens to the graph when you add a constant to the y term in the equation? _____

3.
$y = x^2$
$y = 2x^2$
$y = 5x^2$
$y = \frac{1}{2}x^2$
$y = \frac{1}{6}x^2$

What happens to the graph when you multiply the $x^2$ term by a positive constant? _____

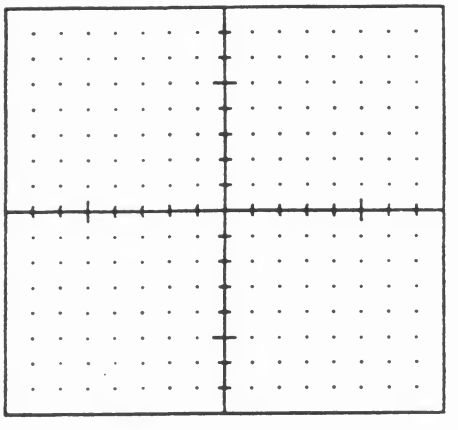
SUNBURST COMMUNICATIONS © 1985

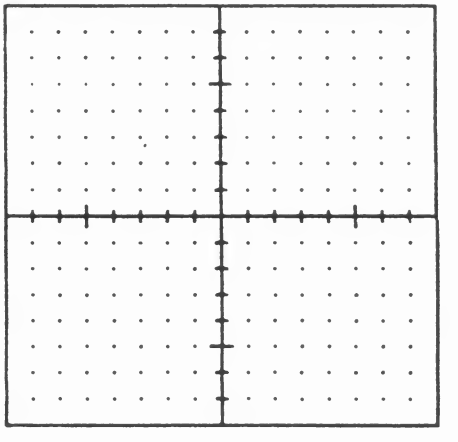
4. $y = x^2$
   $y = -x^2$
   $y = -(x+3)^2$
   $y = -\frac{1}{5}x^2$

   What happens to the graph when you multiply the $x^2$ term by a negative number? _____

5. $x = y^2$
   $x = (y-3)^2$
   $x = 4y^2$
   $x = \frac{1}{2}y^2+4$
   $x = -3y^2$

   What happens to the graph when you reverse the x and the y in the equation? _____

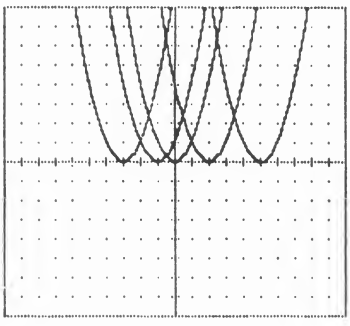




SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #3 - PARABOLAS

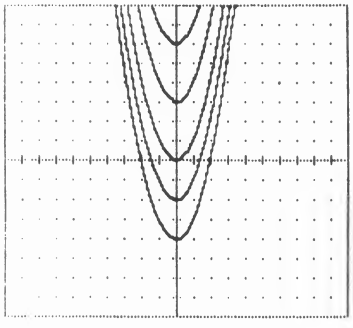
1.



| vertex |
|---|
| 0,0 |
| -1,0 |
| -3,0 |
| 2,0 |
| 5,0 |

the graph moves right or left

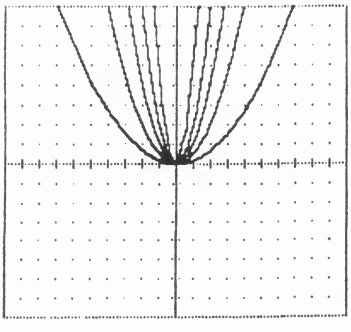
2.



| vertex | touch or cross |
|---|---|
| 0,0 | yes |
| -2,0 | yes (-1½,0)(1½,0) |
| -4,0 | yes (-2,0)(2,0) |
| 3,0 | no |
| 6,0 | no |

the graph moves up or down

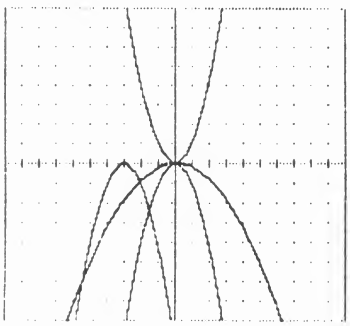
3.



the graph becomes steeper or flatter

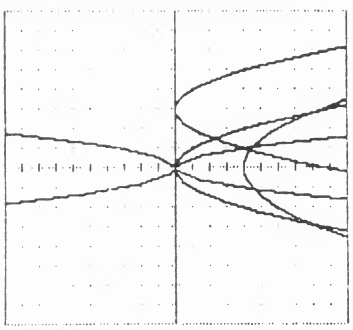
SUNBURST COMMUNICATIONS © 1985

4.



the graph is "upside down," it opens downward

5.



the graph opens to the left or right instead of up or down

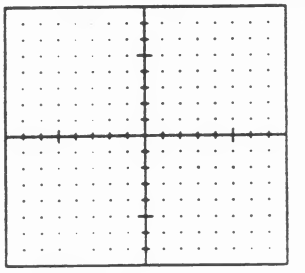
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#4 - GRAPHING CIRCLES

Enter each set of equations on the same coordinate grid. Observe the differences and similarities among the graphs.

Use ESC then RETURN to clear the grid. Use the " < " and " > " keys or the back arrow key ← to save typing.

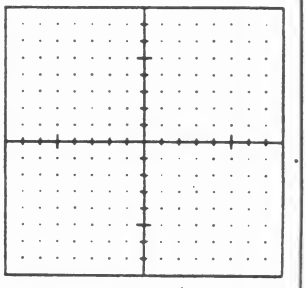
Sketch the graphs.



1. $x^2 + y^2 = 4$
   $x^2 + y^2 = 9$
   $x^2 + y^2 = 16$
   $x^2 + y^2 = 25$

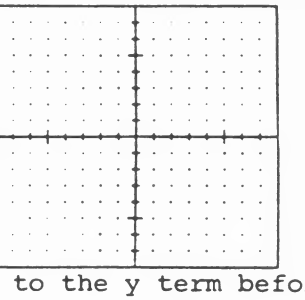
   a. How does the graph of the circle change when the constant term increases? _____

   b. Where is the center of each of these circles? _____



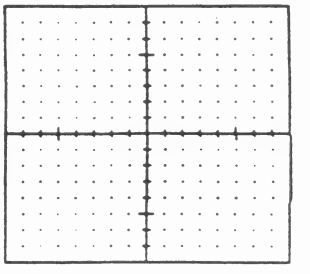
| | | center |
|---|---|---|
| 2. | $(x+1)^2 + y^2 = 16$ | _____ |
| | $(x+3)^2 + y^2 = 16$ | _____ |
| | $(x-4)^2 + y^2 = 16$ | _____ |
| | $(x-6)^2 + y^2 = 16$ | _____ |

How does the graph change when a constant is added to the x term before it is squared? _____



| | | center |
|---|---|---|
| 3. | $x^2 + (y+4)^2 = 25$ | _____ |
| | $x^2 + (y+1)^2 = 25$ | _____ |
| | $x^2 + (y-1)^2 = 25$ | _____ |
| | $x^2 + (y-5)^2 = 25$ | _____ |

How does the graph change when a constant is added to the y term before it is squared? _____

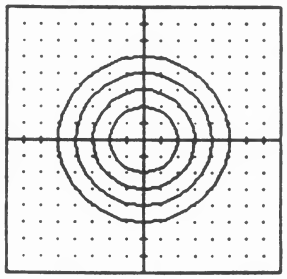


| | | center |
|---|---|---|
| 4. | $(x+1)^2 + (y-2)^2 = 16$ | _____ |
| | $(x-3)^2 + (y-4)^2 = 25$ | _____ |
| | $(x+4)^2 + (y+5)^2 = 36$ | _____ |
| | $(x-2)^2 + (y+3)^2 = 9$ | _____ |

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #4 - GRAPHING CIRCLES

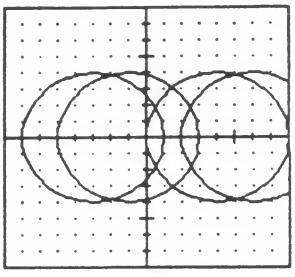
1. $x^2 + y^2 = 4$
   $x^2 + y^2 = 9$
   $x^2 + y^2 = 16$
   $x^2 + y^2 = 25$



   a) the radius increases
   b) 0, 0

2.

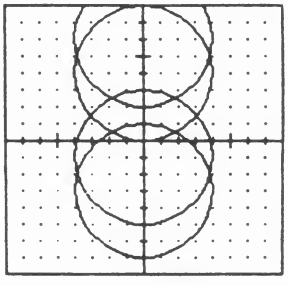
| | center |
|---|---|
| $(x+1)^2 + y^2 = 16$ | -1, 0 |
| $(x+3)^2 + y^2 = 16$ | -3, 0 |
| $(x-4)^2 + y^2 = 16$ | 4, 0 |
| $(x-6)^2 + y^2 = 16$ | 6, 0 |



The center moves right or left (on x axis)

3.

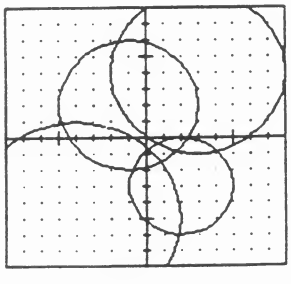
| | center |
|---|---|
| $x^2 + (y+4)^2 = 25$ | 0, -4 |
| $x^2 + (y+1)^2 = 25$ | 0, -1 |
| $x^2 + (y-1)^2 = 25$ | 0, 1 |
| $x^2 + (y-5)^2 = 25$ | 0, 5 |



The center moves up or down (on the y axis)

4.

| | center |
|---|---|
| $(x+1)^2 + (y-2)^2 = 16$ | -1, 2 |
| $(x-3)^2 + (y-4)^2 = 25$ | 3, 4 |
| $(x+4)^2 + (y+5)^2 = 36$ | -4, -5 |
| $(x-2)^2 + (y+3)^2 = 9$ | 2, -3 |



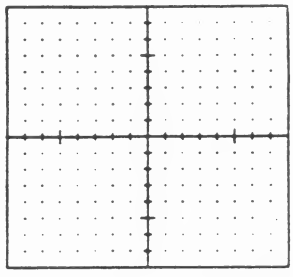
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#5 - GRAPHING ELLIPSES

Enter each set of equations on the same coordinate grid. Observe the differences and similarities among the graphs.

Use ESC then RETURN to clear the grid. Use the " < " and " > " keys or the back arrow key ← to save typing.

Sketch the graphs.



vertices

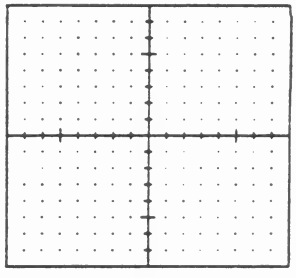
1. $x^2/9 + y^2 = 1$ _____

$x^2/9 + y^2/4 = 1$ _____

$x^2/9 + y^2/9 = 1$ _____

$x^2/9 + y^2/25 = 1$ _____

How does the graph change as the denominator of the y term changes?_____



vertices

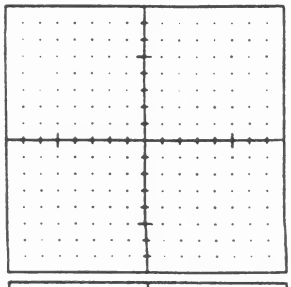
2. $x^2 + y^2/16 = 1$ _____

$x^2/4 + y^2/16 = 1$ _____

$x^2/16 + y^2/16 = 1$ _____

$x^2/25 + y^2/16 = 1$ _____

How does the graph change as the denominator of the x term changes?_____



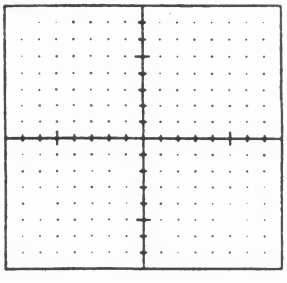
vertices

3. _____

$x^2/9 + y^2/4 = 1$

$(x - 2)^2/9 + y^2/4 = 1$ _____

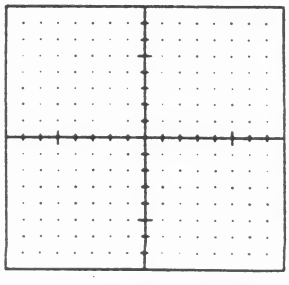
$(x + 3)^2/9 + y^2/4 = 1$ _____



4. _____

$x^2/9 + (y - 4)^2/4 = 1$ _____

$x^2/9 + (y + 5)^2/4 = 1$ _____



5. _____

$(x - 3)^2/16 + (y + 2)^2/25 = 1$ _____

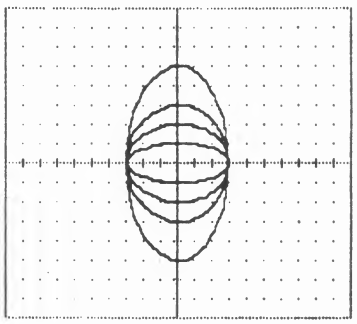
$(x + 4)^2/36 + (y - 3)^2/9 = 1$ _____

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #5 - GRAPHING ELLIPSES

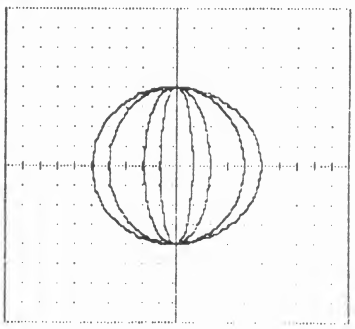
1. vertices
(3,0) (-3,0) (0,1) (0,-1)
(3,0) (-3,0) (0,2) (0,-2)
(3,0) (-3,0) (0,3) (0,-3) (a circle)
(3,0) (-3,0) (0,5) (0,-5)

The graph becomes shorter or taller.
The vertices move farther out along
the y-axis as the denominator increases.

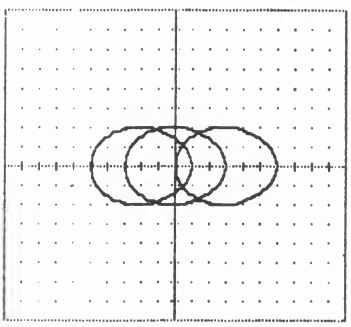


2. (1,0) (-1,0) (0,4) (0,-4)
(2,0) (-2,0) (0,4) (0,-4)
(4,0) (-4,0) (0,4) (0,-4) (circle)
(5,0) (-5,0) (0,4) (0,-4)

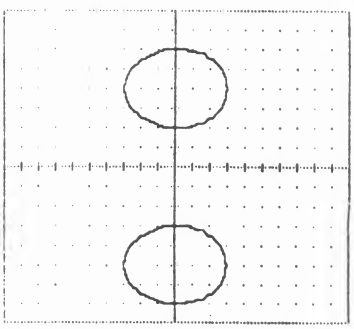
The graph becomes wider or narrower.
The vertices move farther out along the
x-axis as the denominator increases.



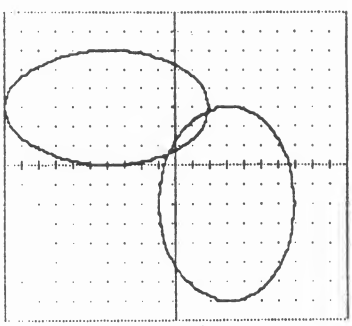
3. (3,0) (-3,0) (0,2) (0,-2)
(5,0) (-1,0) (3,2) (3,-2)
(0,0) (-6,0) (-3,2) (-3,-2)



4. (3,4) (-3,4) (0,2) (0,6)
(3,-5) (-3,-5) (0,-3) (0,-7)



5. (-1,-2) (7,-2) (3,3) 3,-7)
(-10,3) (2,3) (-4,6) (-4,0)



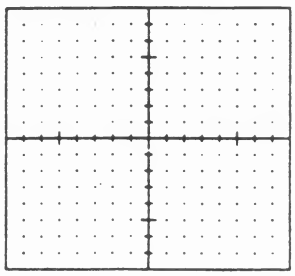
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#6 - GRAPHING HYPERBOLAS

Enter each set of equations on the same coordinate grid. Observe the differences and similarities among the graphs.

Use ESC then RETURN to clear the grid. Use the " < " and " > " keys or the back arrow key ← to save typing.

Sketch the graphs.



vertices

1. $x^2 - y^2 = 1$ _____

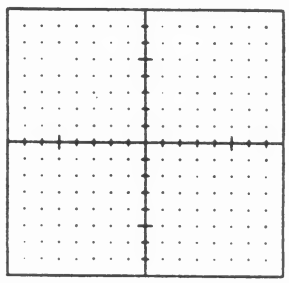
$x^2/4 - y^2 = 1$ _____

$x^2/9 - y^2 = 1$ _____

$x^2/25 - y^2 = 1$ _____

How does the graph change when the denominator of the x term changes?

_____



2. $x^2 - y^2 = 1$ _____

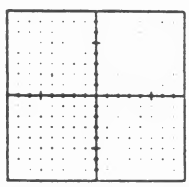
$x^2 - y^2/4 = 1$ _____

$x^2 - y^2/36 = 1$ _____

$x^2 - y^2/64 = 1$ _____

How does the graph change when the denominator of the y term changes?

_____



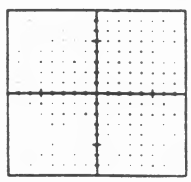
3. $x^2/16 - y^2/25 = 1$ _____

$(x - 3)^2/16 - y^2/25 = 1$ _____

$(x + 4)^2/16 - y^2/25 = 1$ _____

How does the graph change when a constant is added to the x term before squaring?

_____



4. $x^2/16 - (y - 2)^2/25 = 1$ _____

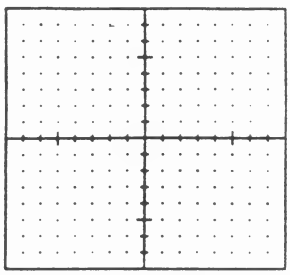
$x^2/16 - (y + 4)^2/25 = 1$ _____

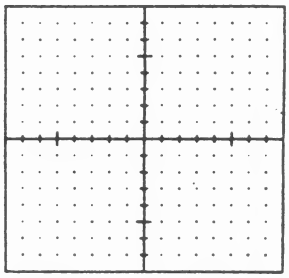
How does the graph change when a constant is added to the y term before squaring? _____

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#6 - GRAPHING HYPERBOLAS - page 2

5. $(x+2)^2/25 - (y-1)^2/16 = 1$ vertices _____





6. $y^2/16 - x^2/9 = 1$ _____

$y^2/9 - x^2/4 = 1$ _____

How does the graph change when the x and y are reversed in the equation?

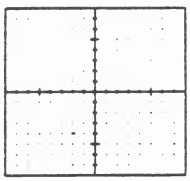
_____

7. The graphs of these equations are also hyperbolas. How are the graphs different from those above? _____

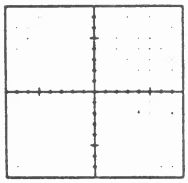
a. $xy = 1$
$xy = 9$
$xy = 25$
b. $xy = -16$
$xy = -4$

_____
_____
_____
_____
_____

a.



b.

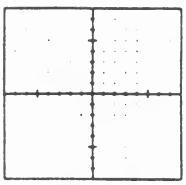


What are the equations of the lines on which the vertices lie?

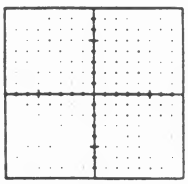
_____

8a. $(x-3)y = 1$
$x(y+4) = 1$
b. $(x+2)y = -1$
$x(y-5) = -1$

a.



b.

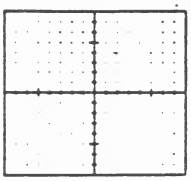


How does the graph change when you add a constant to the x or the y term?

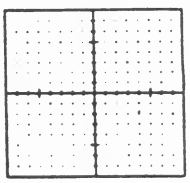
_____

9a. $xy = 4$
$x(y+4) = 4$
$(x-2)y = 4$
b. $xy = -9$
$(x+3)(y-2) = -9$

a.



b.



How does the graph change when you change the constant term?

_____

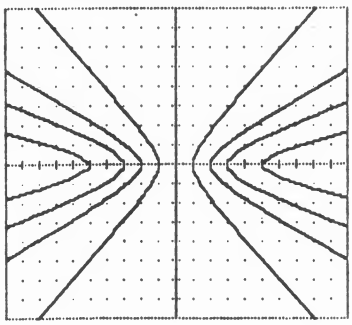
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #6 - GRAPHING HYPERBOLAS

1. (±1,0) or (1,0)(-1,0)
(±2,0) or (2,0)(-2,0)
(±3,0) or (3,0)(-3,0)
(±5,0) or (5,0)(-5,0)

The vertices move farther out along the x-axis as the denominator increases.

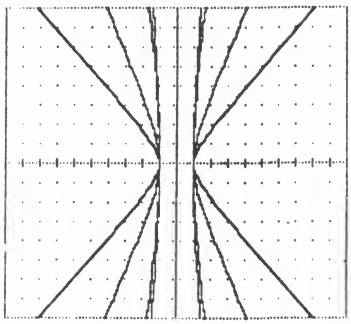
1.



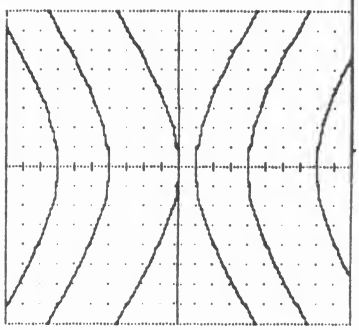
2. All vertices are (±1,0)

The graph becomes more vertical as the denominator increases.

2.



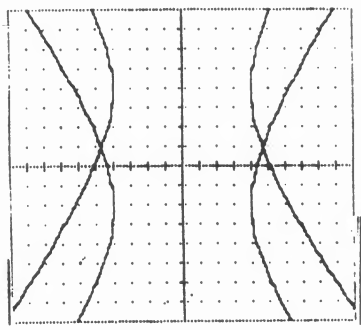
3.



3. (±4,0)
(7,0) (-1,0)
(0,0) (-8,0)

The graph moves right or left.

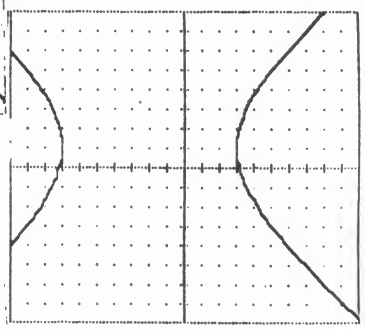
4.



4. (4,2) (-4,2)
(4,-4) (-4,-4)

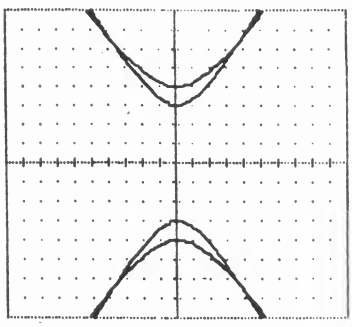
The graph moves up or down.

5.



5. (3,1) (-7,1)

6.



6. (0,±4) or (0,4) (0,-4)
(0,±3) or (0,3) (0,-3)

The graph opens upwards and downwards instead of sideways.

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #6 - GRAPHING HYPERBOLAS - page 2

7. The graphs are tilted; they lie entirely inside the first and third quadrants of the grid.

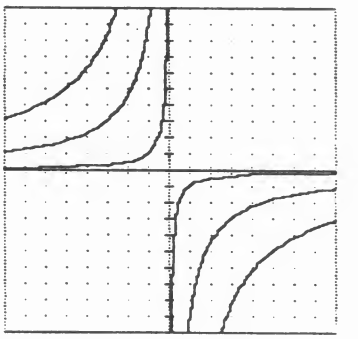
(1,1) (-1,-1)
(3,3) (-3,-3)
(5,5) (-5,-5)

The graphs here lie inside the 2nd and 4th quadrants.

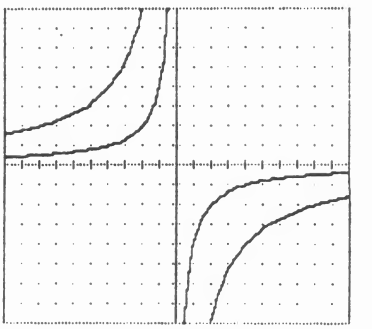
(-4,4) (4,-4)
(-2,2) (2,-2)

The equations of the lines are $y = x$ and $y = -x$.

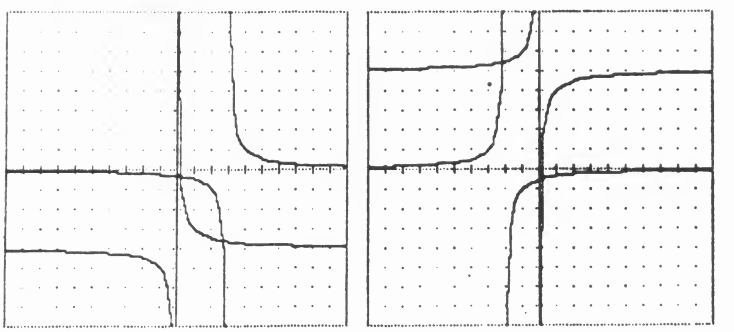
7a.



7b.

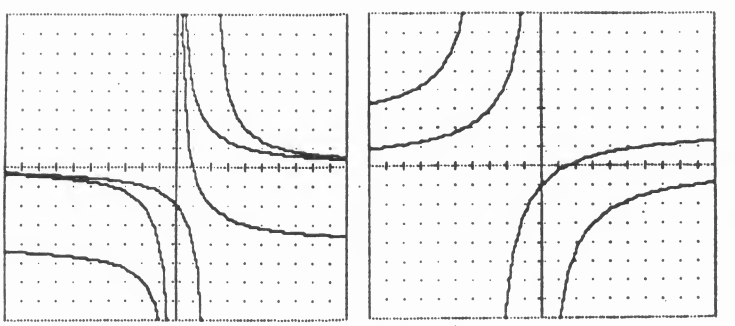


8a. 8b.



The graph is shifted horizontally or vertically.

9a. 9b.



Changing the size of the constant term changes the curvature of the graph and the distance between its vertices. Changing the sign of the constant term reflects the graph with respect to an asymptote.

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#7 - EXPLORING ROOTS

Use the program EQUATION PLOTTER to plot the graphs of the equations in the exercises below.

A. Plot the graph of $y = x^2 - 4x + 3$.
Where does it cross the x axis (what are its roots)? _____

Factor the right side of the equation, $y = x^2 - 4x + 3$
$y = (\quad)(\quad)$

Compare the factors with the roots. What do you notice? _____
_____

B. Plot the graph of $y = x^2 - 3x - 10$.
Where does it cross the x axis? _____

Factor the right side of the equation. $y = (\quad)(\quad)$
Compare the factors with the roots. _____

1. Write an equation of a parabola that crosses the x axis at the points (-2,0) and (2,0). Plot it and see if you are correct.
_____

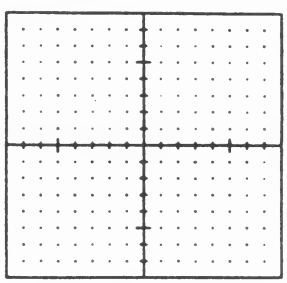
2. Write an equation of a parabola that crosses the x axis at the points (2,0) and (6,0). Plot it and see if you are correct.
_____

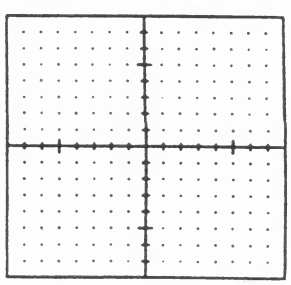
3. Write an equation of a parabola that crosses the x axis at (-5,0) and (1,0). _____

4. Write an equation of a parabola that crosses the x axis at (1.5,0) and (4.5,0). _____

5. Write an equation of a parabola that passes through the points (-4,3) and (4,3). _____

6. Write an equation of a parabola that passes through the points (2,-4) and (6,-4). _____

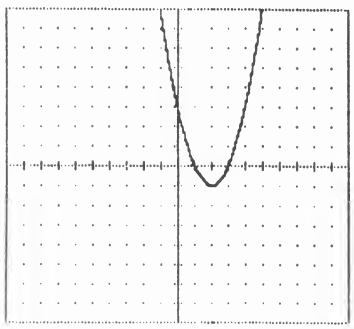




SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #7 - EXPLORING ROOTS

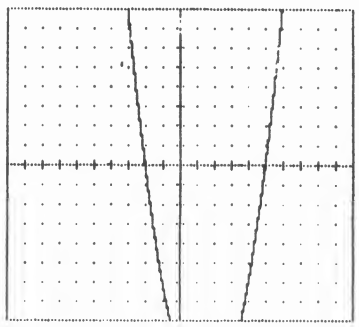
A.



roots (+1,0) (3,0)
y = (x-1)(x-3)

For each root R, there is a factor (x-R).

B.



roots (5,0) (-2,0)
y = (x-5)(x+2)

For each root R, there is a factor (x-R).

1. $y = c(x+2)(x-2)$ or $y = x^2-4$

2. $y = c(x-2)(x-6)$ or $y = x^2-8x+12$

3. $y = c(x+5)(x-1)$ or $y = x^2+4x-5$

4. $y = c(x-1.5)(x-4.5)$ or $y = x^2-6x+6.75$

5. $y = c(x+4)(x-4) + 3$ or $y = x^2-13$

6. $y = c(x-2)(x-6)-4$ or $y = (x^2-8x+8)$

In the above equations, c can be any non-zero constant.

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#8 - PASSING THROUGH POINTS

1. Write an equation whose graph crosses the x axis at (-4,0) and (5,0) and that passes through the point (3,-7).

First, locate these points on a grid.

Clearly, a straight line will not pass through these three points, so consider a second degree equation.

Using the divide and conquer strategy, consider only the points on the x axis, x=-4 and x=5. At these points y equals zero.

Using guess and check, create two factors to multiply together and set them equal to y. The factors must be chosen so that when x=-4 or 5, the product is zero.

y = (  )(  )

Check that this graph does cross the x axis at -4 and 5 by entering it into the EQUATION PLOTTER program and observing the graph.

(If your first try did not work, modify the equation on the basis of what you see and try again.)

Look to see if this graph passes through the other point, (3,-7). It probably does not.

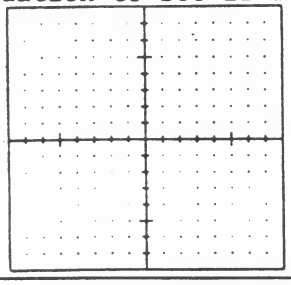
The graph must pass through (3,-7). First substitute 3 for x in the equation above, to find the value of the equation at x=3.

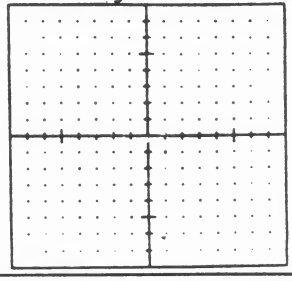
y = (x  )(x  )
y = (3  )(3  )
y = (  )(  )
y = -14

The value of y must be -7, not -14. Just as you need to divide -14 by 2 to get -7, you can divide the equation by 2 to get the new equation.

y = (x+4)(x-5)/2

Graph this equation to see if it passes through all three points.





SUNBURST COMMUNICATIONS © 1985

Now change this equation so that its graph crosses the x axis at the same points, but passes through the point shown below instead of the point (3,-7):

a. (3, +28) y = _____

b. (3, -21) y = _____

c. (1, -10) y = _____

d. (1, 5) y = _____

2. Write an equation whose graph crosses the x axis at (-2,0) and (3,0) and passes through the point (-1,-1).

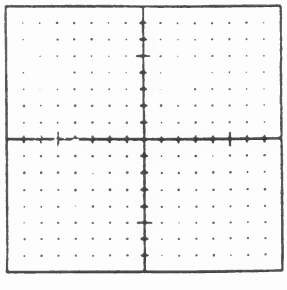
y = _____

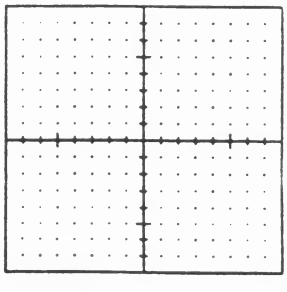
3. Write an equation whose graph crosses the y axis at (0,-1) and (0,3) and passes through the point (6,0).

x = _____

4. Write an equation whose graph crosses the x axis at (2,0), (-2,0), (-5,0) and passes through the point (-4,6).

y = _____

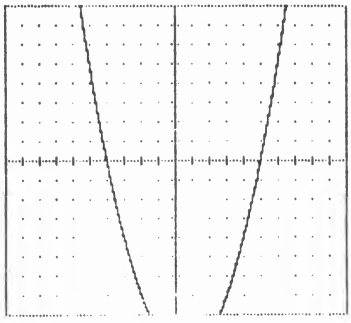




SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER

ANSWERS - #8 - PASSING THROUGH POINTS



1. $y = (x+4)(x-5)$
   $y = (3+4)(3-5)$
   $= (7)(-2)$
   $= -14$

a. $y = -2(x+4)(x-5)$

b. $y = \frac{3}{2}(x+4)(x-5)$

c. $y = \frac{1}{2}(x+4)(x-5)$

d. $y = -\frac{1}{4}(x+4)(x-5)$

2. $y = (x+2)(x-3)/4$ or $y = \frac{1}{4}(x+2)(x-3)$

3. $x = -2(y+1)(y-3)$

4. $y = \frac{1}{2}(x-2)(x+2)(x+5)$

SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
# 9 - ADVANCED TECHNIQUES:
Sums of Functions

This activity demonstrates the effect of adding a term of the form 1/x to a function.

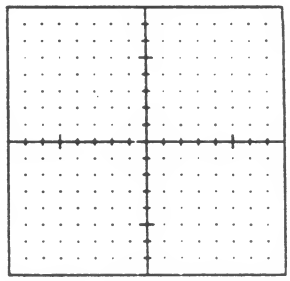
1. Using the EQUATION PLOTTER program, graph the following rational functions:

   $y = 1/x$, $y = 1/(x-2)$, and $y = 1/(x+4)$

   Observe the results. What happens at the values x=0 in the first graph, x=2 in the second graph, and x=-4 in the third?_____
   What happens to the denominator in each equation at these values of x?

   _____

2a. Using EQUATION PLOTTER and a blank grid, graph the equation $y = 1/(x-2)$. Sketch the graph on the grid at the right.



b. Recall the graph of the line y=x. Graph it on the computer if you wish. Sketch its graph on the grid at the right.

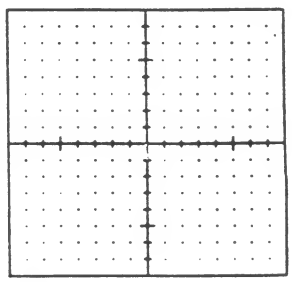
c. Now use the computer to graph the equation formed by adding the term 1/(x-2) to the equation of the line.

   $y = x + 1/(x-2)$

   Describe the graph. _____

   _____

   Sketch the graph.



3a. Graph the equation $y = 1/(x-1)$. Sketch the graph.

b. Graph the parabola $y = x^2/4$. Sketch the graph.

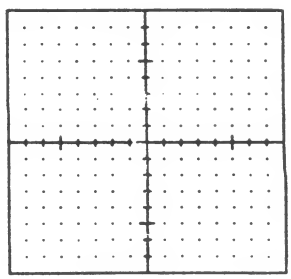
c. Use the computer to graph the equation formed by adding these two terms.

   $y = x^2/4 + 1/(x-1)$

   Describe the graph._____

   _____

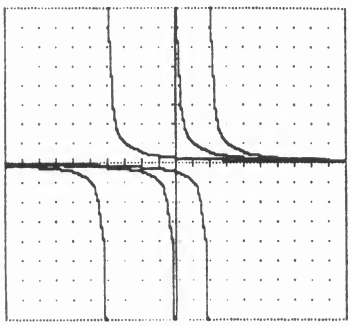
   Sketch the graph.



SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #9 - SUMS OF FUNCTIONS

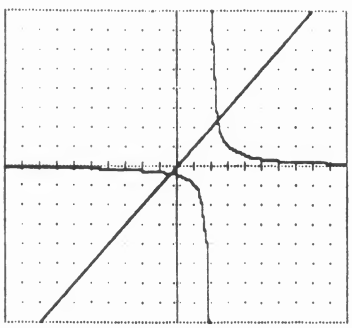
1.

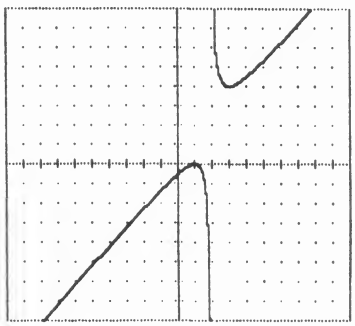


The graph is not defined, does not exist.

The denominator is zero.

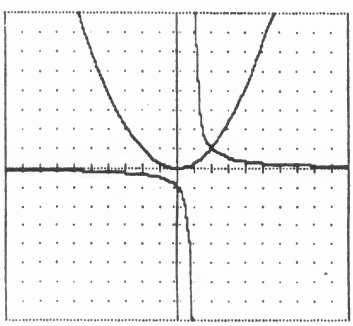
2.

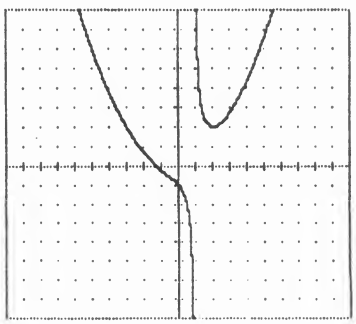




It looks like y=x except near x=2 where it steeply turns downwards, then from the top curves sharply down and continues almost along y=x.

3.





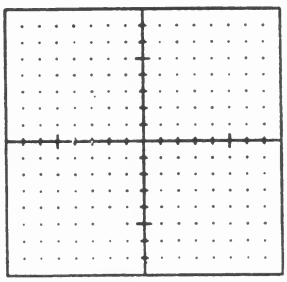
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#10 - ADVANCED TECHNIQUES:
Trigonometric Functions

Graph each set of equations on the same coordinate grid. Notice the similarities and differences. Sketch the graphs.

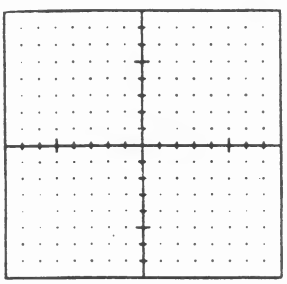
The ideas you get from these activities may help you play the Expert Level of GREEN GLOBS.

1. Graph y = SIN(x)
   y = 5SIN(x)
   y = SIN(5x)



   a. What is the effect of putting a coefficient in front of the SIN function?_____

   b. What is the effect of putting a coefficient in front of the x inside the SIN function?_____
   _____

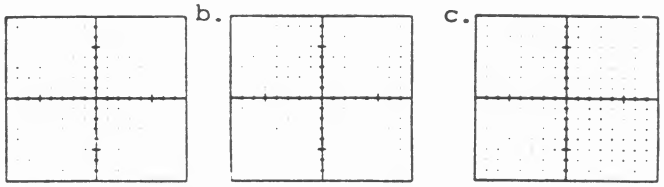


2. Graph x = SIN(y)
   x = 6SIN(y)
   x = SIN(4y)

   What differences are there between the graphs in this set and those above? _____

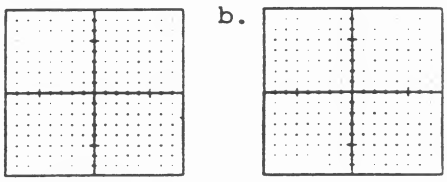
3. Graph
   a. y = TAN(x)
   b. y = TAN(x)/8
   c. y = TAN(3x)

a. b. c.



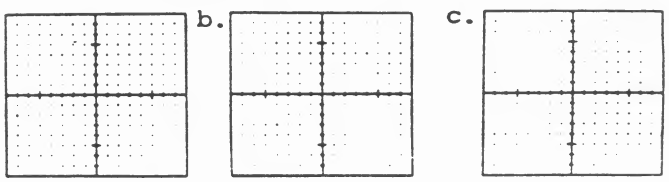
4. Graph
   a. x = TAN(y)
   b. x = TAN(5y)/2

a. b.



5. Graph
   a. y = SEC(x)
   b. y = SEC(3x)
   c. x = SEC(y)

a. b. c.

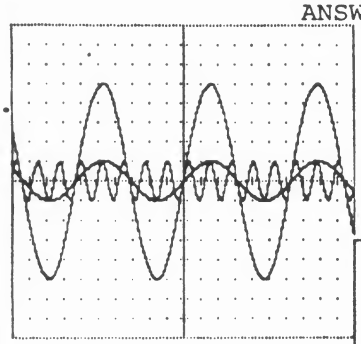


SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER

ANSWERS - #10 - TRIGONOMETRIC FUNCTIONS

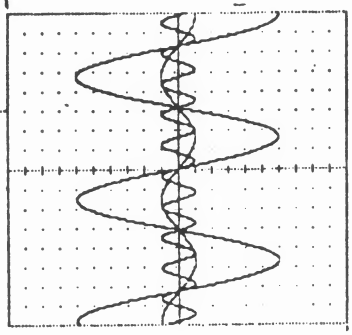
1.



a. The height of the graph is increased; it is stretched up and down.

b. The period is smaller; the graph is squeezed together sideways.

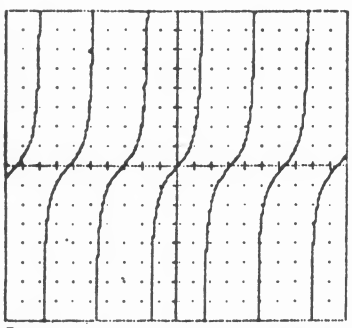
2.



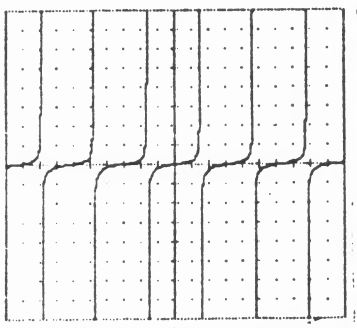
These graphs lie along the y-axis rather than the x-axis.

3.

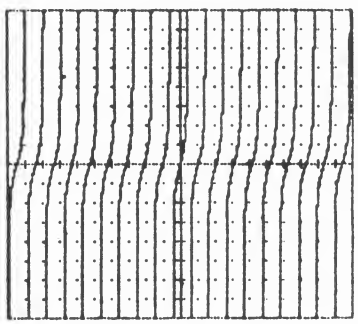
a.



b.

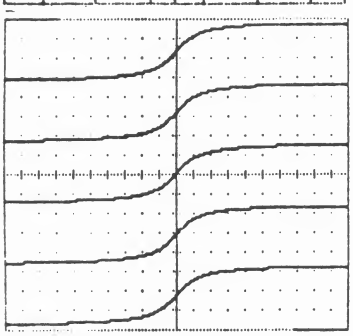


c.

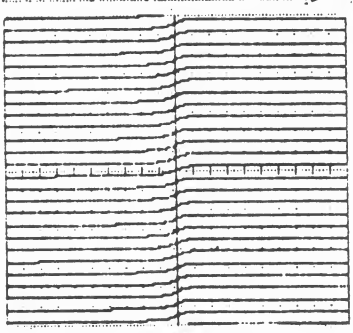


4.

a.

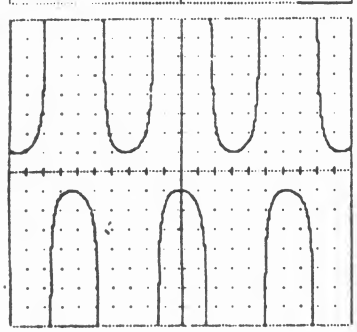


b.

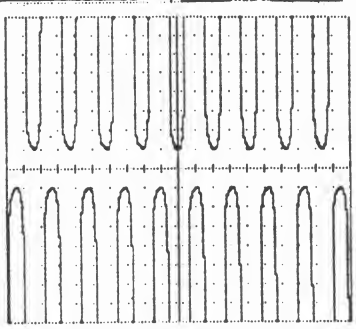


5.

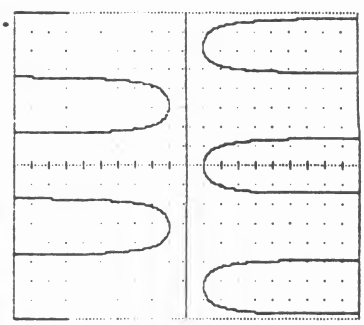
a.



b.



c.



SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#11 - ADVANCED TECHNIQUES:
Sums with Trigonometric Functions

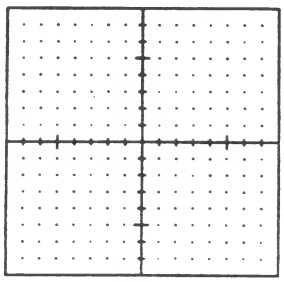
Interesting graphs can be produced by adding two functions. Try these examples of sums of functions, then create some combinations of your own.

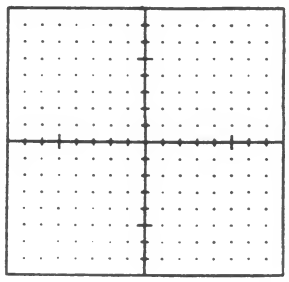
Graph the equations on the computer with EQUATION PLOTTER, then sketch the graphs on paper so you'll have a record of them. The ideas you get from these sum equations may prove helpful when you play the Expert Level of GREEN GLOBS.

1. Graph $y = x$
   $y = SIN(x)$
   and $y = x + SIN(x)$

   Describe the graph of the sum equation $y = x + SIN(x)$._____

   _____





2. Graph $y = -x/2$
   and $y = -x/2 + 3SIN(x)$

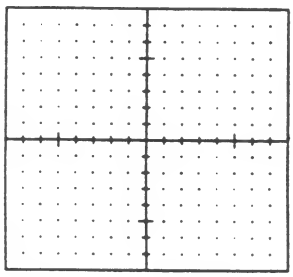
   Describe the graph._____

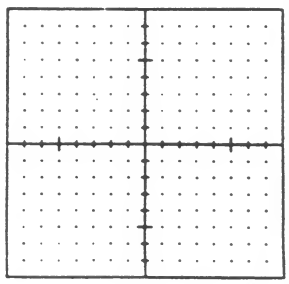
   _____

3. Graph $x = y$
   $x = y + 4SIN(y)$

   Describe the graph._____

   _____





4. Graph $y = 2x - 3$
   $y = 2x - 3 + 4SIN(3x)$

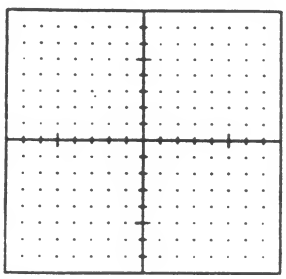
   Describe the graph._____

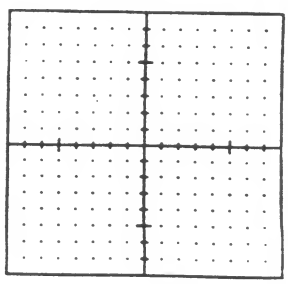
   _____

5. Graph $y = x^2/4$
   $y = x^2/4 + SIN(4x)$

   Describe the graph._____

   _____



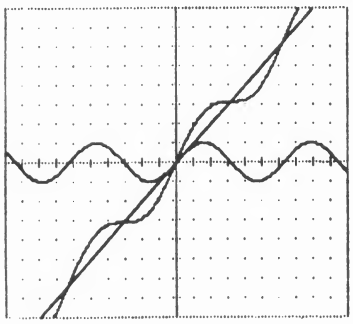


6. Try using COS(x) and other variations of the above equations

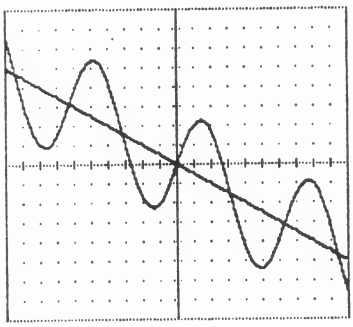
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER

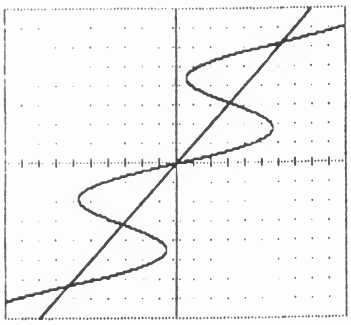
ANSWERS - #11 - SUMS OF TRIGONOMETRIC FUNCTIONS

1. 

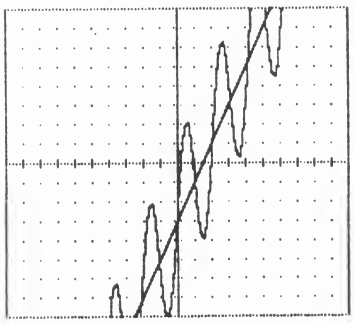
**The graph is a slightly wavy curve along the path of y = x.**

2. 

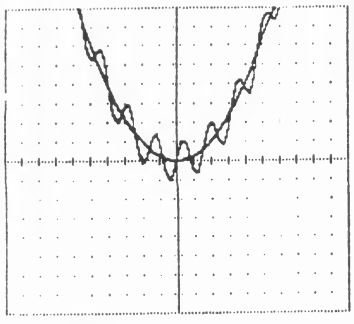
The graph is a bumpy curve along the path of y = - x/2

3. 

The graph is a very wavy curve along the path of x = y.

4. 

The graph is a very sharp curve along the path of y = 2x-3.

5. 

The graph is a wavy curve along the path of $y = x^2/4$.

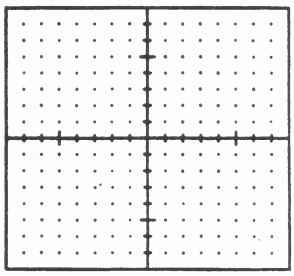
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
#12 - ADVANCED TECHNIQUES:
Absolute Value Functions

The absolute value function can be combined with other functions to produce interesting effects. Try these examples, then create innovative graphs of your own.

Graph the equations on the computer using EQUATION PLOTTER, then sketch the graphs on paper, so you'll have a record of them to refer to when you play GREEN GLOBS.

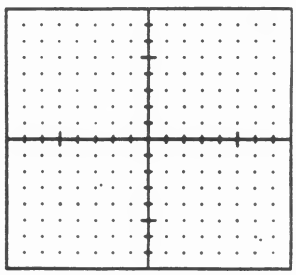
1. Sketch the graphs of y = x and y = ABS(x).



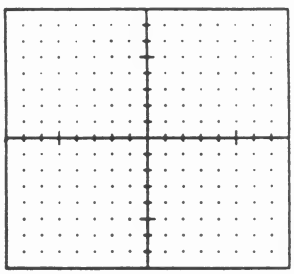
2. Use the computer to graph these equations:

   y = 2x -3
   y = ABS(2x-3)
   y = ABS(2x) - 3

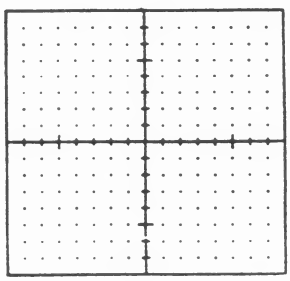
   Sketch the graphs. Describe each graph._____
   _____



3. Graph x = ABS(y) and x = ABS(y/4). Sketch the graphs. Describe them.
   _____



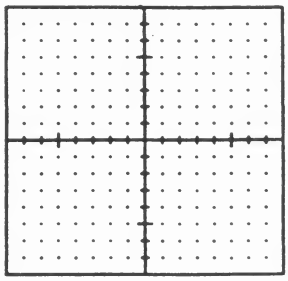
4. You know the graph of $y = x^2$. What does the graph of $y = ABS(x^2)$ look like?
   _____
   (not too exciting, is it!)



SUNBURST COMMUNICATIONS © 1985

5a. Graph $y = x^2 - 5$, then graph $y = ABS(x^2 - 5)$.
Describe the graphs._____
_____

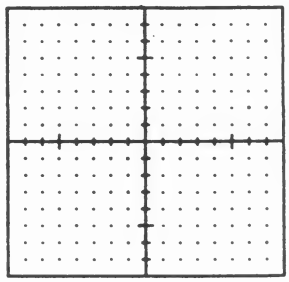
b. Graph $y = ABS(x^2 - 5) - 7$.
Describe the graph._____
_____



6. Try using the absolute value function with trigonometric functions.

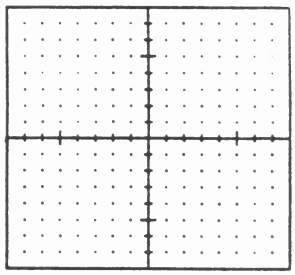
a. Graph $y = TAN(x)$
$y = ABS(TAN(x))$

b. Graph $x = TAN(y)$
$x = ABS(TAN(y))$



7. Try adding the absolute value function to another function.

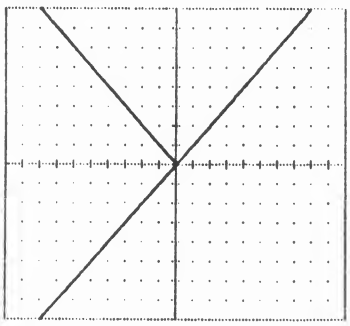
Graph $y = ABS(3TAN(x)) + x$



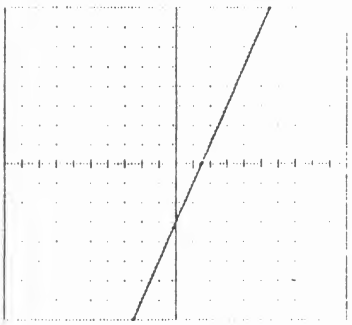
SUNBURST COMMUNICATIONS © 1985

EQUATION PLOTTER
ANSWERS - #12 - ABSOLUTE VALUE FUNCTIONS

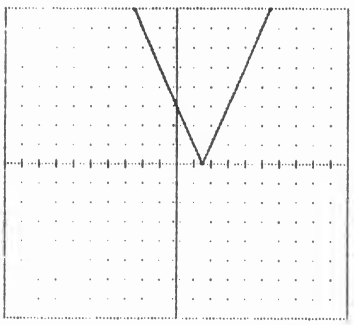
1. y = x
   y = ABS(x)



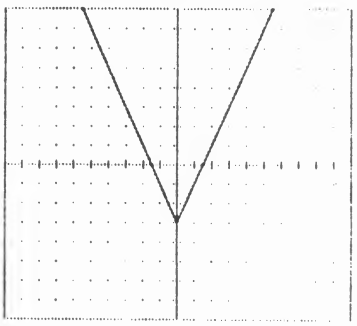
2. y = 2x-3
   straight line



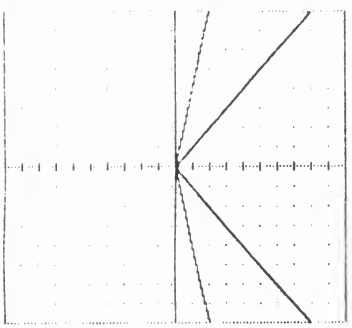
y = ABS(2x-3)
a V-shaped graph



y = ABS(2x)-3
V-shaped graph



3. x = ABS(y/4)

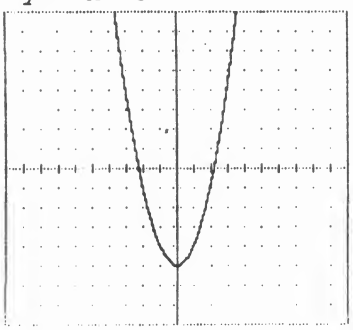


The graphs are V-shaped
and open to the right

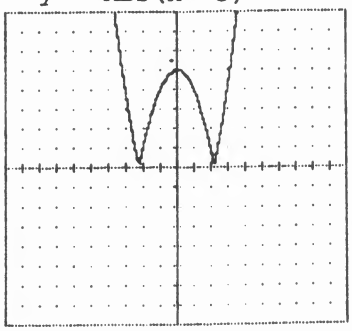
4. Same graph as $y = x^2$

SUNBURST COMMUNICATIONS © 1985

5a. $y = x^2-5$

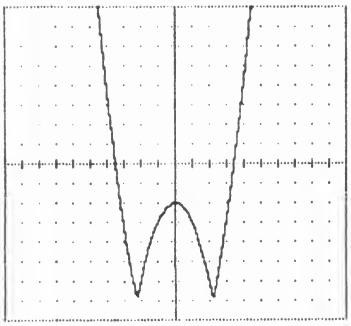


$y = ABS(x^2-5)$



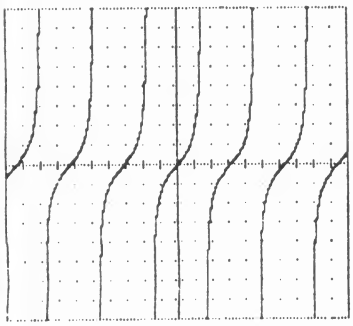
The negative part of the parabola has been flipped around the x-axis.

b.

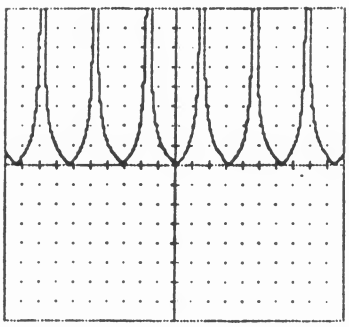


$y = ABS(x^2-5)-7$
same shape as $y = ABS(x^2-5)$, but 7 lower

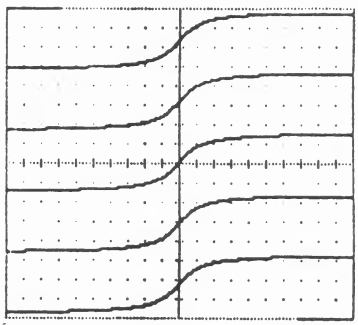
6a. $y = TAN(x)$



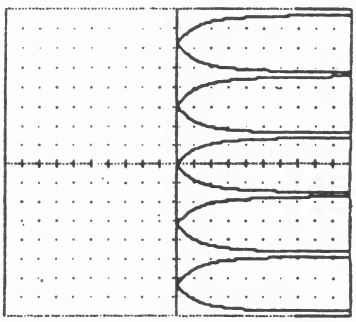
$y = ABS(TAN(x))$



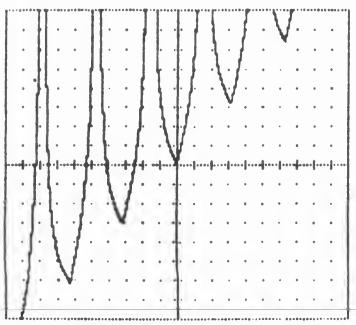
b. $x = TAN(y)$



$x = ABS(TAN(y))$

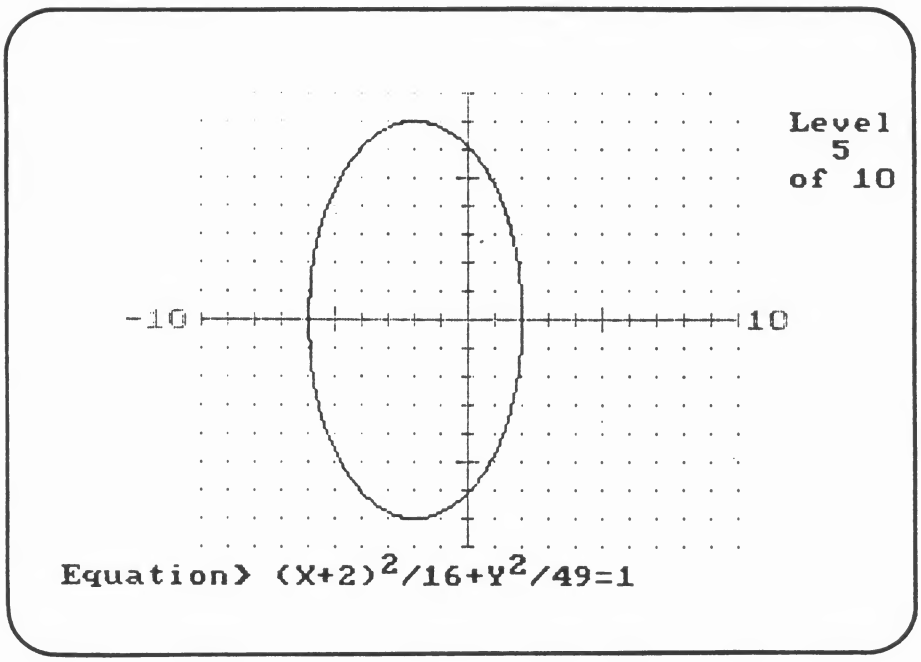


7. $y = ABS(3TAN(x))+x$



SUNBURST COMMUNICATIONS © 1985

LINEAR AND QUADRATIC GRAPHS




Grade Level: 8 - 14

Prerequisite Skills: an introduction to linear and quadratic graphs

Type: practice in relating equations to graphs

Description

Students choose the type of linear or quadratic graph to be used: line, parabola, circle, ellipse, hyperbola. A graph is displayed on a coordinate grid. Students enter equations to match the target graph shown.

The graph of each equation entered is plotted on the screen in a different color from the target graph. Students continue entering equations and observing their graphs until the equation matches the target graph.

Objectives

1. to recognize basic shapes of linear and quadratic graphs
2. to relate graphs to their equations
3. to translate graphs horizontally and vertically
4. to manipulate graphs by altering the corresponding equation
5. to generalize transformation concepts and strategies from simple to more complex graphs
6. to use trial and error to diagnose errors and revise hypotheses

PROGRAM DESCRIPTION

Students choose from five types of graphs:

- straight lines
- parabolas
- circles
- ellipses
- hyperbolas (two forms)
- mixture of all types

For each type of graph, there is a sequence of difficulty levels. The student can choose the following options:

1. begin with easy problems (level 1)
2. begin with more difficult problems (choose any level)
3. work problems of mixed difficulty levels (mixed levels)

The sequence for each type of graph begins with the basic shape. Either ten or twenty levels of difficulty are available for each graph type. In options 1 and 2, the student will automatically advance to the next level each time he/she matches a target graph on the first try.

Most levels include several similar graphs, but a few levels contain only one graph. For example, Level 6 for parabolas includes the graphs

$y = -(x-1)^2$, $y = -(x-2)^2$, and so on,

but Level 1 is always the graph $y = x^2$.

See #13--Instructions and Tips (page 45) for details on using this program.

IDEAS FOR CLASSROOM USE

Before Students Use the Program

Students should have seen the basic equations and corresponding graphs of the types they will use on the computer. For example, the program section on parabolas should accompany or follow a unit of study on parabolas, and students should be familiar with the equation $y=x^2$ and its graph.

Use the EQUATION PLOTTER program to play "what if" for the type of equations under study. For example, graph the equation $y=x^2$, then ask "what if there was a minus sign before the $x^2$ term?" Graph $y=-x^2$ and observe the results. Another example would ask "what if x was replaced by (x+2)?" Graph this equation and observe the changes.

Demonstrate LINEAR AND QUADRATIC GRAPHS to the class, using example problems such as those on the sample demonstration sheet on page 43.

Distribute copies of #13--Instructions and Tips to the students.

Using the Program

Encourage students to begin with easy problems, to use trial and error problem solving, and to persevere.

Point out that "mistakes" can be valuable learning experiences in this activity. Often the graph of an incorrect equation provides important clues to the correct equation. Discuss the information obtained by comparing its graph to the target graph.

Remind students that their goal is to develop general strategies that can be applied to many types of graphs, rather than memorize formulas.

Ask students to use the #14--Recording Sheet (page 46) to keep track of their progress.

Follow-up

Encourage students to generalize their results. How does the equation change if the graph--

is moved up or down on the grid?
is moved to the right or left?
is "upside down" or slanting down?
is steeper or flatter?

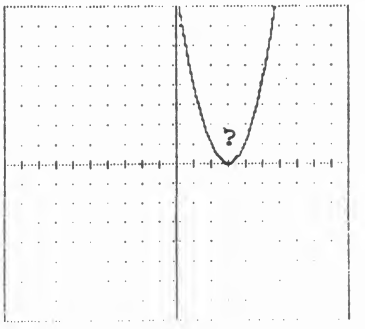
Play GREEN GLOBS, Novice game.

Play TRACKER using the type of graphs practiced in this program.

Sample Classroom Demonstration on Parabolas

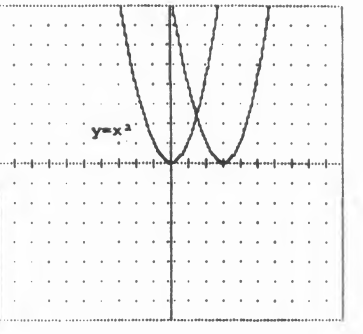
Note that your examples may differ slightly from these. Be open to suggestions and ideas from the students, even if they might take you off the track of the planned demonstration.

1. Here is a graph of some equation. We need to figure out what the equation is.




We know it is a parabola, so we can start with the basic equation for a parabola, $y=x^2$.

Let's graph that equation for comparison.

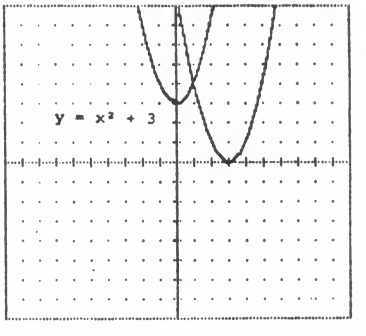



Looking at the graphs we see that the target graph is shifted to the right 3 units from the origin.

Let's experiment. What if we added 3 to the equation?

$$y = x^2 + 3$$

Let's graph it and see.

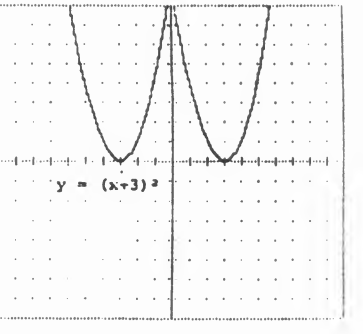



Well, that's not what we want. It moved the parabola up, not to the right. What else could we try?

Let's try using x+3 in place of x

$$y = (x+3)^2$$

and graph that.

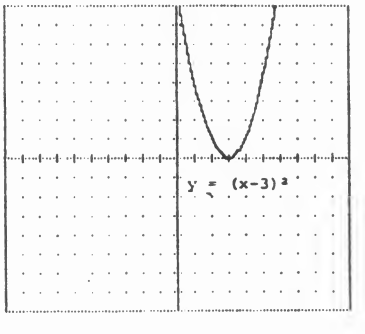



That's not it either! This time the graph is shifted to the left by 3.

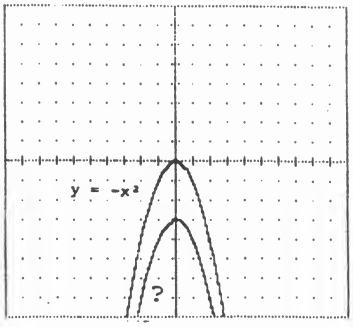
The form of the equation is right but the direction is wrong, so let's change from +3 to -3.

$$y = (x-3)^2$$

That works!



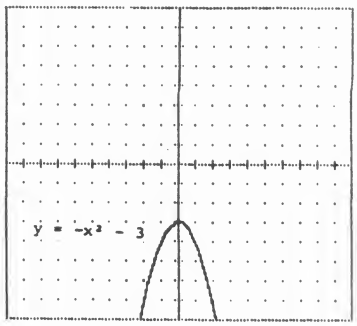
2. Here's another graph. We know it's a parabola, but it's upside down (inverted).




What can we change in the equation to invert the graph? Try changing from + to -:

$y = -x^2$

That works. Now how can we move that inverted parabola down three units?

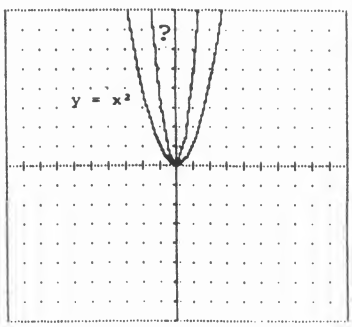



We need to subtract 3 from every x value, so try

$y = -x^2 - 3$

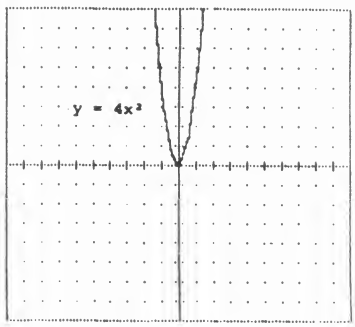
Graph it. That's it!

3. Here's another parabola, so we'll start with $y = x^2$ again.




Its vertex is at the origin but it's higher at each x value than the basic parabola.

It appears to be 4 times higher at each x value (e.g. (1,4) is on the graph instead of (1,1)).

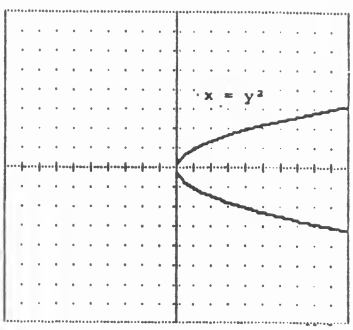



Let's try multiplying the function by 4.

$y = 4x^2$

That's it.

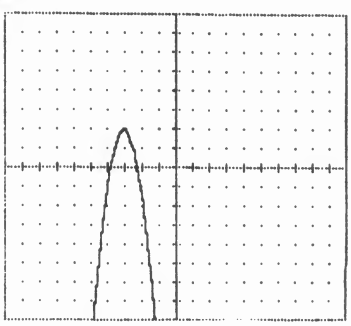
4. Here's a parabola that is "going sideways." How could we change the basic equation to match this graph?




Switch the x and the y.

$x = y^2$

5. How could we combine several of these ideas to find the equation of almost any parabola?



LINEAR AND QUADRATIC GRAPHS
#13 - INSTRUCTIONS AND TIPS

A graph is displayed on a coordinate grid. Your job is to enter an equation that matches that graph. The equation you enter is plotted in a different color on the same grid. Compare your graph with the original graph, analyze the differences, and modify your equation as needed.

You may enter as many equations as you like until you match the original graph.

You can choose the type of graph:
- straight line
- parabola
- circle
- ellipse
- hyperbola
- a mixture of your choice

You also choose
1. to begin at the easiest problems
2. to begin at more challenging problems, or
3. to work problems of mixed difficulty levels.

Start with option 1. If you leave the program before finishing, record your current level on your recording sheet. The next time you use the program, choose option 2 and start at the level you recorded.

You will advance to the next level of difficulty when you match the target graph on your first try without help.

If you get stuck on a problem, you can have the program show you the matching equation. Of course, this prevents you from moving to the next level on this problem.

Tips for Success

Use these problem-solving strategies:

Simplify the Problem - Start with the basic equation of the type of graph (for example, $y=x^2$ for a parabola).

Divide and Conquer - Break the problem down into smaller parts. If the graph is shifted, is inverted, and is stretched or squeezed, change the basic equation for each of these one at a time.

Guess and Check - Make your best guess for the equation, then carefully compare its graph and the target graph. Then improve your equation to make a better guess on your next try.

DO NOT try to memorize formulas for every graph!

SUNBURST COMMUNICATIONS © 1985

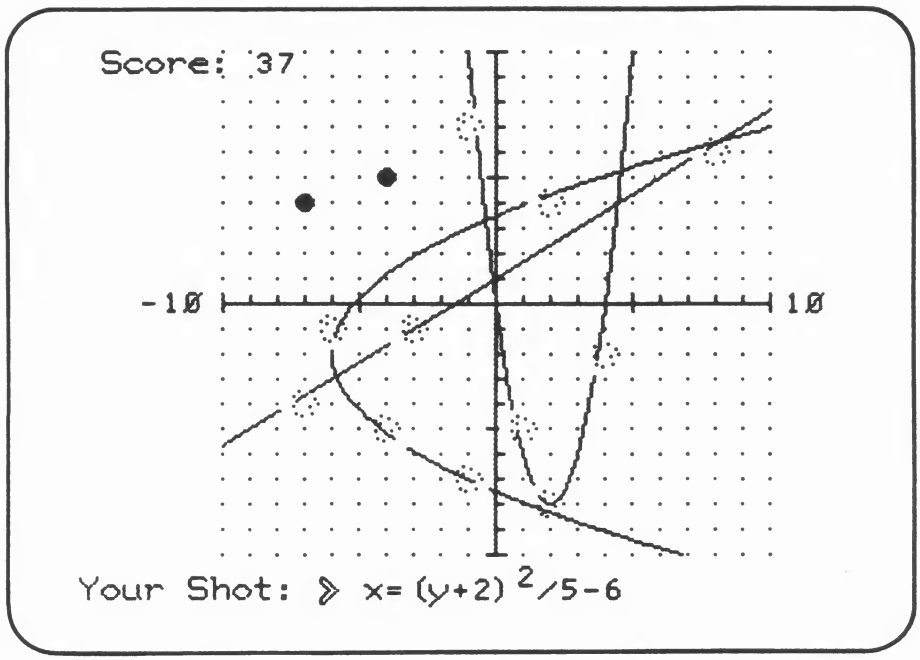
LINEAR AND QUADRATIC GRAPHS
#14 - RECORDING SHEET

Shade the squares to show the levels you complete.

| Type of Graph Used | Level Completed |
|---|---|
| straight line | 1 2 3 4 5 6 7 8 9 10<br>11 12 13 14 15 16 17 18 19 20 |
| parabola | 1 2 3 4 5 6 7 8 9 10<br>11 12 13 14 15 16 17 18 19 20 |
| circle | 1 2 3 4 5 6 7 8 9 10 |
| ellipse | 1 2 3 4 5 6 7 8 9 10 |
| hyperbola<br>like $x^2 - y^2 = 1$ | 1 2 3 4 5 6 7 8 9 10 |
| hyperbola<br>like $xy = 1$ | 1 2 3 4 5 6 7 8 9 10 |
| mixed graphs | number of problems completed _____ |

SUNBURST COMMUNICATIONS © 1985

GREEN GLOBS




Grade Level: 8 - adult

Prerequisite Skills: introduction to graphing equations

Type: strategy game using graphing concepts.

Description

Thirteen randomly scattered "green globs" are displayed on a coordinate grid. The students' goal is to explode all the globs by hitting them with the graphs of equations entered at the keyboard. The scoring algorithm encourages students to hit as many globs as possible with each equation. The top ten scores are recorded along with the games that produced them. Students can review these games to learn strategies to apply to future games.

Objectives

1. to apply skills in relating equations to their graphs
2. to construct equations to fit specified criteria
3. to discover and explore relationships between equations and their graphs
4. to create strategies to maximize the number of hits per equation
5. to transform basic graphs by modifying their equations
6. to work cooperatively in a group
7. to explore types of equations not found in standard texts
8. to strengthen skills in problem solving: guess and check, simplify, and divide and conquer
9. to apply graphically a wide range of algebraic concepts and techniques

PROGRAM DESCRIPTION

## Novice Game

The coordinate grid displays thirteen randomly scattered green globs. Students enter equations for graphs that will hit (pass through) these globs. Each glob is 0.7 units in diameter. Touching any part of a glob is sufficient to explode it.

When a glob is hit, it explodes and disappears. If the graph misses the target globs, the display of the graph provides useful diagnostic information for determining the next equation to try.

This visual feedback enables students to use the guess and check strategy. They improve their next equation on the basis of information gained from their previous "shot."

Students of widely varying mathematical ability can effectively use this program. Clearly it is possible to hit all the globs with only linear equations. Beginning algebra students can use linear equations and benefit from playing GREEN GLOBS. Advanced students can use more sophisticated equations to earn higher scores.

The scoring algorithm encourages students to hit as many globs as possible with each equation.

## Scoring Algorithm

For each graph, the first glob hit is worth one point, the second is worth two points, the third is worth four points, and so on. Thus a five-glob shot scores 1 + 2 + 4 + 8 + 16 for a total of 31 points.

This doubling algorithm means that hitting one more glob on any shot approximately doubles the score.

## Types of Equations

Students can enter functions beginning with "y=" or "x=".

For example: $y = x^2-3x+2$
$x = (y+2)(y-1)^3$
$y = 3+x+5/x-(x-1)^3/(x^2-x)$

Equations of conics are also accepted.

For example: $x^2+5y^2 = 30$
$(x-4)^2/10 - (y+2)^2/2 = 1$
$x^2=6, \quad xy=5$

Functions using square roots, absolute value, logarithmic and exponential functions are also accepted.

Students may NOT use trigonometric functions in the Novice Game. This is to prevent graphs like y = 10SIN(10x) from hitting all the globs with one shot.

## Sharing Strategies

The top ten scores and the names of those students for the Novice Game are kept in the program. The games that produced these top scores are also stored, so they can be studied by other students. By viewing these games, students learn strategies and gain new ideas that they can apply to future games they play.

There is no danger of students "cheating" or avoiding thinking by looking at the stored games. The random placement of globs on each game ensures that using other students' strategies requires more than simply copying their equations. To use a strategy, students must understand how it relates to the locations of the globs, and they must apply it to the new situation.

Teachers may want to erase the current top ten scores and games when using this program with a new group of students. This is particularly recommended when the experience level of the new class is substantially different from that of the previous class. To remove the top ten scores, use Control-T along with your password, as described in detail on page 5.

Worksheets #7 and 8 in EQUATION PLOTTER suggest some strategies that are useful in GREEN GLOBS. It is recommended that students be given opportunities to develop their own strategies before being given worksheets.

Worksheets #9 through 12 in EQUATION PLOTTER explore advanced types of equations, such as trigonometric functions, sums of functions, and absolute value equations. These are designed for use by students who have had considerable experience in using GREEN GLOBS.

## Expert Game

The Expert Game differs from the Novice Game in two ways:

1. In addition to the thirteen green globs, the coordinate grid contains five "shot absorbers" that stop any shots that hit them.

2. Students are allowed to enter trigonometric functions, as well as all of the equations that can be used in the Novice Game. For example,

$$y = SIN(x)$$
$$y = 5SEC(2x)-3x+1$$
$$y = (x^2/5)(COS(3x))$$
$$x = 1/(TAN(1/y^2))$$

   are all acceptable in the Expert Game.

The equations recognized in the Expert Game are the same as those allowed in EQUATION PLOTTER. They include the following:

- lines, parabolas, circles, ellipses, hyperbolas where x and y appear only once in the equation,

- equations where y equals a function of x,

- equations where x equals a function of y,

- trigonometric functions, SIN, COS, TAN, CSC, SEC, COT, ATN
  square roots, SQR
  absolute value, ABS
  natural logarithms, LOG
  exponential functions, EXP.

The top ten scores and games for the Expert Game are kept separate from the Novice Game scores.

See #15--Instructions and Tips (page 55) for more detailed information on using this program.

IDEAS FOR CLASSROOM USE

Before Using the Program

Students should have had practice in writing equations for graphs. The program LINEAR AND QUADRATIC GRAPHS and the utility EQUATION PLOTTER may be used as preparatory or concurrent activities. The TRACKER game also provides useful experience in manipulating equations and graphs.

The teacher should understand that GREEN GLOBS is a challenging game that may require a considerable amount of time and mathematical experimentation. They should allow students to play the game over a period of days or weeks.

GREEN GLOBS can be fascinating and highly rewarding. Students of all mathematical ability levels can gain satisfaction and new insights. Students will strengthen their problem-solving skills as well as learn significant mathematical concepts.

Students are encouraged to share their ideas with each other. Reviewing the top ten games stored on the diskette is one way to share discoveries. Working in groups of two or three students can also increase the exchange of ideas.

Using the Program

Groups of two or three students often work well. However, some students, especially the more advanced students, may prefer to work independently.

Students may want to use graph paper to label glob coordinates and intercepts or to sketch ideas. It is useful to identify the coordinates of globs and intercepts rather than just estimating them. Students using sophisticated equations may find a calculator helpful.

Students should be encouraged to develop their own strategies. Teachers should NOT routinely give students strategies to try. Students should be encouraged to share their ideas and to build on and apply others' ideas to their own situations.

Worksheets #7 and 8 in the EQUATION PLOTTER section of this guide provide ideas that may be useful in GREEN GLOBS. Worksheets #9 through 12 present several advanced techniques and are intended for students who have had considerable experience with GREEN GLOBS.

The following are examples of strategies students have developed using GREEN GLOBS:

General Math (9th Grade)

Students used horizontal and vertical lines. The first strategy was to use lines that would hit two globs when possible and to be careful not to hit either of these globs singly with an earlier shot.

After viewing top score games done by algebra students, they began experimenting with curved graphs using equations like those they saw in the top score games.

## Algebra--Using Roots of Equations

Students who can factor binomials and relate the factors to roots of the equations can discover that the graph of

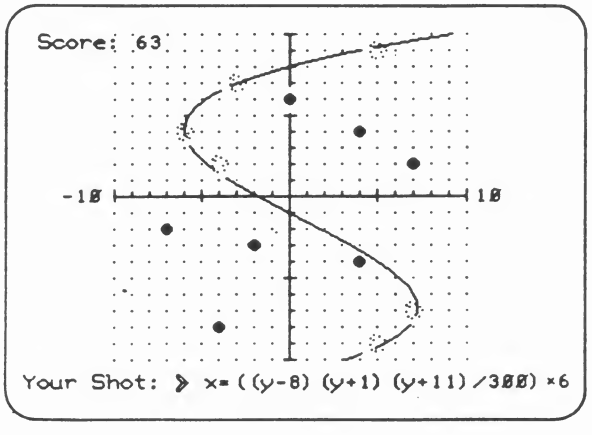
$y = (x+5)(x-2)$ or $y = x^2 + 3x - 10$

will cross the x axis at -5 and 2. They may be able to extend this idea to higher order polynomial equations, such as

$y = (x+5)(x-2)(x-7)$.

EQUATION PLOTTER Worksheet #7 provides discovery activities and practice with this strategy.

The following example builds a polynomial from roots, then uses coordinates of a target glob to figure appropriate coefficients for the equation. EQUATION PLOTTER Worksheet #8 guides students through this process.




Write the equation of a graph that will cross the y axis at three points: 8, -1, -11. $x = (y-8)(y+1)(y+11)$

Use the glob at (-6,4) to figure a scaling factor. Let y = 4.

$$x = (4-89)(4+1)(4+11)$$
$$= (-4)\ (5)\ (15)$$
$$= -300$$

But x must be -6, not -300. So divide the equation by 300 and multiply it by 6 (or simply divide by 50).

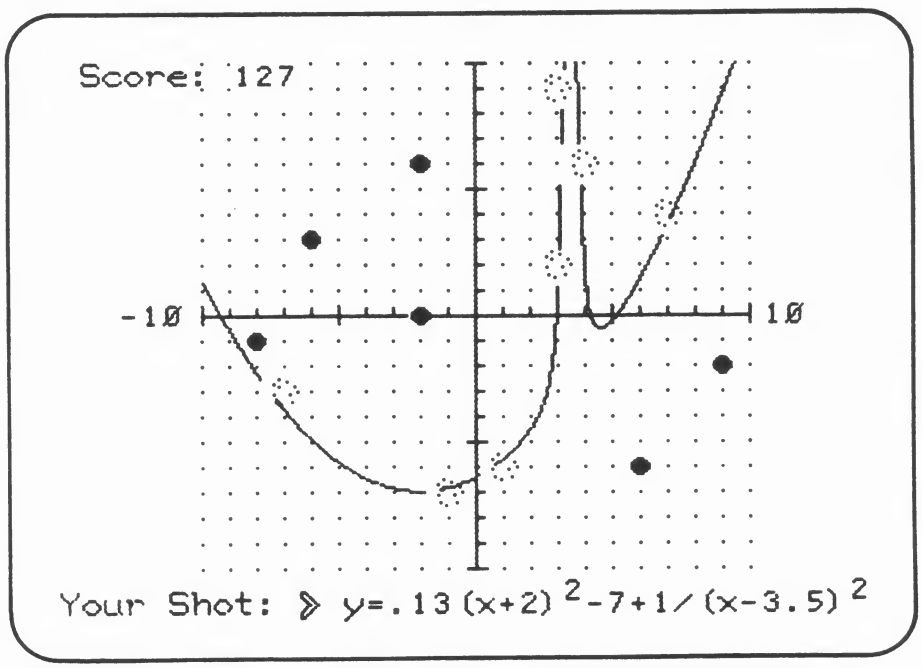
$$x = ((y-8)(y+1)(y+11))/300*6$$

This strategy may lead to the creating of "super polynomials." These are equations built in factored form with many roots and a numerical factor included to stretch the graph. If the numerical factor is large enough, the parts of the graph crossing the screen are a set of nearly vertical (or horizontal) lines.

There is a limit to the space available on the screen for entering the equation. Only about four factors will fit, so super polynomials need to be carefully constructed.

Math Analysis--Adding Terms of the Form 1/(x-a)

Adding a term of the form 1/(x-a) to a function creates a discontinuity at x=a. While students at the pre-calculus level may not have studied discontinuities, they can see through experimentation that the graph of the equation rises (or drops) sharply to the left and right of the point (a,0). Thus the graph may hit globs that are almost above (or below) that point.




Note that for most values of x, the term 1/(x-a) is very small, so it has a negligible effect on the graph, except where x is close to a.

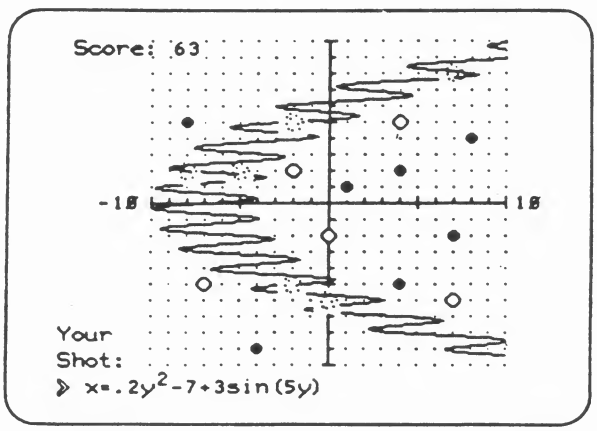
Interesting graphs can result from adding several of these fractional forms together to produce discontinuities at many points.

EQUATION PLOTTER Worksheet #9 provides discovery activities and practice in using this strategy.

## Math Analysis - Sums of Functions

Although most textbooks do not emphasize sums of functions except simple cases such as y = x + SIN(x), the computer's calculating power makes these types of equations easy to explore. The Expert Game allows the use of trigonometric functions.

Here is an example of the sum of a parabola and a sine function. The effect of adding the sine function is to make the graph cover a wider path, thereby hitting more globs, while still avoiding the "shot absorbers."

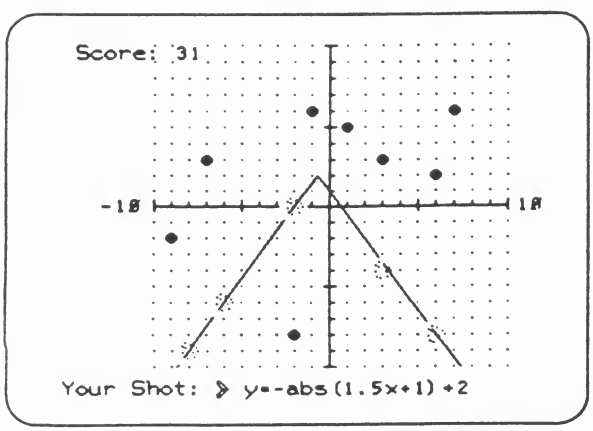


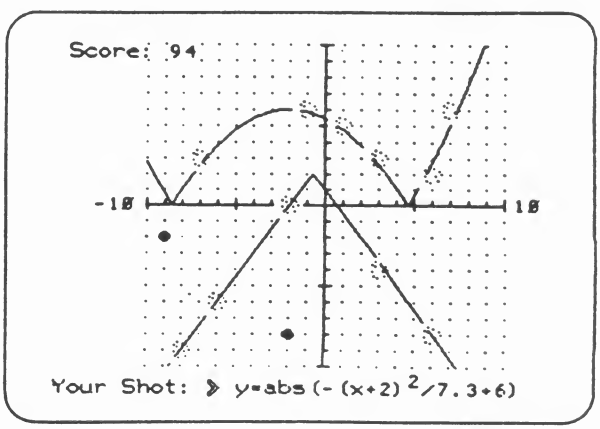

EQUATION PLOTTER Worksheet #11 provides discovery activities and practice in using this strategy.

## Math Analysis - Absolute Value Functions

Combining the absolute value function with other functions can provide a useful strategy.

For example:




EQUATION PLOTTER Worksheet #12 provides discovery activities and practice in using this strategy.

GREEN GLOBS
#15 - INSTRUCTIONS AND TIPS

GREEN GLOBS is a strategy game using graphs of equations. Thirteen randomly scattered "green globs" are displayed on a coordinate grid.

Your goal is to explode all of the globs. The globs explode when they are hit by the graph of an equation that passes through them. You enter equations at the keyboard.

Levels of Difficulty

The Novice Game is the best place to start. You can NOT use trigonometric functions in this game.

The Expert Game is more difficult. You can use trigonometric functions, but there are five "shot absorbers" displayed on the grid. The shot absorbers stop the graph of any equation that hits them.

Scoring

For each equation you enter, the first glob hit is worth one point, the second glob is worth two points, the third is worth four points, the fourth is worth eight points, and so on.

For example, if you hit four globs with one equation, you score is 1 + 2 + 4 + 8 = 15 points.

Equations

Try to choose equations that hit as many globs as possible.

You may use equations of the following types:

- straight lines
- functions beginning with y= or x=
- equations for conics (such as circles, hyperbolas)
- functions using square roots or absolute value
- equations using logarithmic or exponential functions

Trigonometric functions can be used ONLY in the Expert Game!

Top Scores and Games

In both the Novice and Expert Games, the top ten scores and the games that produced them are stored on the diskette. After you try a few games of GREEN GLOBS, you may want to review the Top Ten Games to see strategies other students have used.

The green globs are RANDOMLY placed on the grid. Although they can be hit with lines and other simple graphs, the more creative you are, the more globs you can hit with each shot!

SUNBURST COMMUNICATIONS © 1985

GREEN GLOBS
#16 - RECORDING SHEET

On the grid below, sketch the green globs.

Label the globs' coordinates.

Record your successful equations.

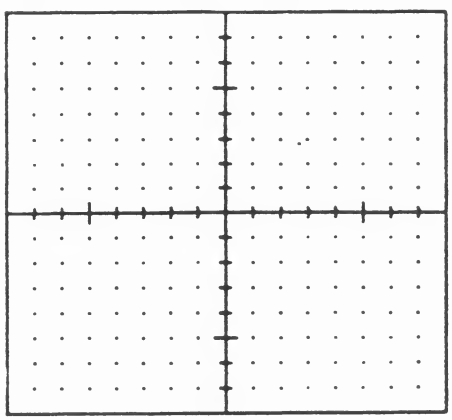
_____
_____
_____

Sketch the graphs of successful equations.

Briefly describe the strategies you used.

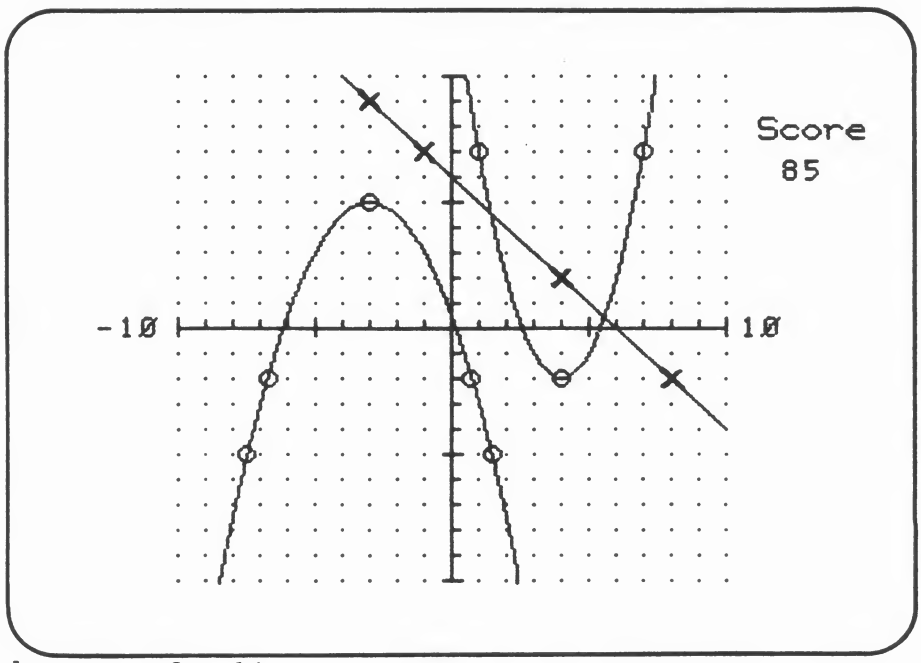
_____
_____
_____
_____

Your score _____



SUNBURST COMMUNICATIONS © 1985

TRACKER




Grade Level: 8 - 14

Prerequisite Skills: introduction to linear and quadratic graphs, ability to write equations to match graphs of lines and conics

Type: game using strategy and graphing concepts

Description

In the TRACKER game, graphs are hidden on a coordinate grid. Students gather clues about the location of the graphs by using "probe shots." These probe shots are horizontal and vertical lines. Each point where the probe crosses the hidden graphs is marked on the screen.

After using probes to locate a graph, students send a "tracker" along the graph by entering its equation.

The Novice Game and the Expert Game provide two levels of challenge.

Objectives

1. to relate graphs to their equations
2. to apply transformation concepts to graphs of equations
3. to develop efficient strategies for locating useful points on hidden graphs
4. to generalize strategies for use with many types of graphs
5. to analyze characteristics of various types of graphs
6. to apply geometric properties, such as symmetry

PROGRAM DESCRIPTION

## General

In TRACKER, the types of hidden graphs are selected from the following types:

- straight line
- parabola
- circle
- ellipse
- hyperbola

The number of hidden graphs varies, but there are never more than two types of graphs in any game.

Students use two kinds of "shots" in the game: probes and trackers. Probes are equations of horizontal or vertical lines entered by students. They are used to find clues about the hidden graphs. As a probe crosses the screen, it marks each point where it intersects a hidden graph. Graphs of one type are marked with an X, and graphs of the other type are marked with an O. A hidden graph is never a horizontal or vertical line.

Depending on where a probe is sent, it may hit several hidden graphs or none at all. Probes mark only those graphs that have not yet been tracked.

After using probes to locate a graph, students send a tracker along the graph by entering its equation. The program accepts equations that are very close to the hidden graph as "close enough." At any time, students can switch between using probes and trackers. If students choose to forfeit a game, the program displays any remaining hidden graphs.

## Practice Section

In the Practice Section, students choose the type of graph that is hidden and whether one or two graphs are hidden. In this way students can concentrate on strategies for locating just one hidden graph. No score is kept in this section.

## Novice Game

In the Novice Game, students choose the two types of graphs hidden on the grid. There are always three hidden graphs: two of one type and one of the other. One type of graph is marked with X's and the other with O's when crossed by a probe shot.

## Expert Game

In the Expert Game, the program randomly chooses the two types of graphs, so students do not know what types of graphs are hidden. Students must use probes to determine the types of graphs as well as their locations. Students decide how many graphs will be hidden--from three to six.

Although higher scores are possible when there are more hidden graphs, it becomes very challenging to sort out more than three or four graphs on the screen. In fact, if probes and trackers are not used carefully and sparingly, a screen with five or six hidden graphs can become too cluttered to complete the game.

## Game Scoring

The scoring algorithm encourages students to minimize the number of probes and trackers used.

| action | points lost |
|---|---|
| first probe | 1 point |
| second probe | 2 points |
| third probe | 3 points |
| ...and so forth | ... |
| incorrect tracker | 10 points |

The points scored for a correct tracker depend on the type of graph.

| graph type | points scored |
|---|---|
| straight line | 30 points |
| parabola | 35 points |
| circle | 40 points |
| ellipse | 45 points |
| hyperbola | 50 points |

The top ten scores are stored in the records. Scores are kept separately for Novice and Expert Games. Games that are forfeited and scores of zero or less are not eligible for the records.

Teachers may want to delete the recorded scores before starting to use this program with a new group of students. This is particularly recommended when the experience level of the new class is substantially different from that of the previous class.

To delete the top ten scores, use Control-T along with your password. This procedure is explained in more detail on page 5.

See Worksheet #17--Instructions and Tips (page 62) for more details on using this program.

IDEAS FOR CLASSROOM USE

Before Using the Program

The EQUATION PLOTTER program can provide introductory activities in graphs of equations. Worksheets #1 through 5 in the EQUATION PLOTTER section of this manual are preparation for playing TRACKER.

The LINEAR AND QUADRATIC GRAPHS program is recommended preparation for playing TRACKER. The graphs used in TRACKER are chosen from the same pool as those in LINEAR AND QUADRATIC GRAPHS. The evaluation of equations is the same, i.e., an equation judged "close enough" in one program is also counted "close enough" in the other.

Distribute copies of TRACKER Worksheet #17--Instructions and Tips. This sheet explains how to play TRACKER and describes the scoring system.

Demonstrate one Practice Section example and one Novice game of TRACKER with the class, so that students understand the process of using probes and trackers. Use their ideas in the demonstration. Emphasize the guess and check strategy and point out the advantage of carefully analyzing the information gained from probes and any incorrect trackers.

Use the TRACKER Sheet #18--Sample Problem in addition to the demonstration or as a substitute if necessary. It provides an introduction to the kind of problem-solving thinking students use in playing the game.

Using the Program

Distribute copies of the TRACKER Recording Sheet #19 for students to use to record their activities.

Encourage students to plan strategies and to share and reuse successful strategies. After students have played the game several times, discuss their strategies in either large or small groups.

If students seem to become discouraged, subtly aid them by asking relevant questions to help them find a new approach.

Here are some examples of the kinds of strategies students may develop. Avoid telling students these or other strategies. The problem-solving process of developing strategies is a most important part of the activity.

> If the sides of a parabola have been located, how can the vertex be found? Try using a probe halfway between the sides, along the axis of symmetry.
>
> What are the key points of a circle? If a probe has crossed a circle, what additional probes are necessary to determine the center? Does it matter whether some of the circle is off the screen?

Is the slope of a line more accurately computed from points that are close together or far apart?

Are four points sufficient to determine two straight lines?

Remind students to apply the transformation skills used in LINEAR AND QUADRATIC GRAPHS; to start with the basic equation and modify it as needed.

If students need to end a game before they find all the hidden graphs, they can forfeit by pressing ESC. They can then see the equations and the graphs they have not found by pressing the SPACE BAR.

Follow-up

Play the GREEN GLOBS game. Random "green globs" appear on the screen. Students try to find equations that pass through the globs. Since the globs are randomly placed, students may want to use other types of graphs in addition to the conic section graphs used in TRACKER.

TRACKER
#17 - INSTRUCTIONS AND TIPS

TRACKER is an equation-graphing game. Graphs are hidden on a coordinate grid. You enter their equations to "track" them.

You get clues to the graphs' locations by using "probe shots." Probe shots are equations of horizontal or vertical lines. When a probe shot crosses a hidden graph, it marks that point with an X or an O.

The types of graphs you choose include the following:

- straight line
- parabola
- circle
- ellipse
- hyperbola

There are three levels of difficulty:

Practice Section is a good place to start. You choose the type of hidden graph. Either one or two graphs are hidden on the grid. No score is kept.

Novice Game allows you to choose two types of graphs. Three graphs are hidden on the grid (two of one type and one of the other). One type of graph is marked by X's and the other by O's when hit by a probe. When you have enough clues, you send a tracker along the graph by entering its equation.

Expert Game chooses the two types of graphs at random, so you'll need to discover what type they are as well as where they are! You choose how many graphs are hidden, three through six.

Scoring

You score points for correctly tracking each graph:

| graph type | points scored |
|---|---|
| straight line | 30 points |
| parabola | 35 points |
| circle | 40 points |
| ellipse | 45 points |
| hyperbola | 50 points |

You lose points for each probe and for incorrect trackers:

| action | points lost |
|---|---|
| first probe | 1 point |
| second probe | 2 points |
| third probe | 3 points |
| ...and so forth | |
| incorrect tracker | 10 points |

The top ten scores are stored in the records. Scores are kept separately for Novice and Expert Games. Games that are forfeited and scores of zero or less are not eligible for the records.

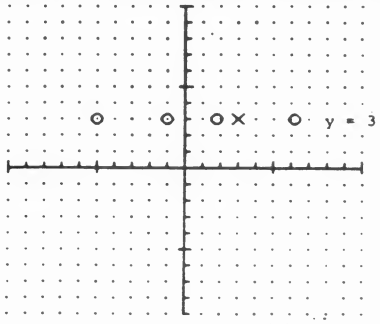
SUNBURST COMMUNICATIONS © 1985

TRACKER
#18 - SAMPLE PROBLEM

Here's an example of one type of problem you might encounter in TRACKER. The questions are the kind you will want to ask yourself when you work similar problems.

You have chosen parabolas and straight lines. There are three graphs in all.

The first probe is
y = 3.
It hits graphs at several points. One type of graph is marked with X, the other type with O.

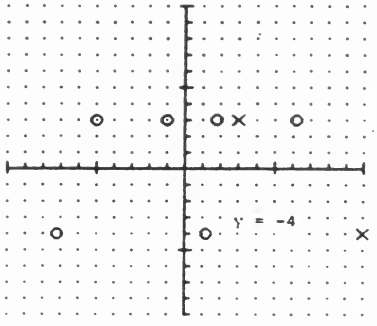



Which type is marked with O in this game? (Since the hidden line(s) cannot be horizontal in this game, it must be parabolas that are marked with O.)

Does the parabola(s) open up or down? Which two points do they pass through?

Try another probe. Where should it be? It could be another horizontal line either above or below y = 3.

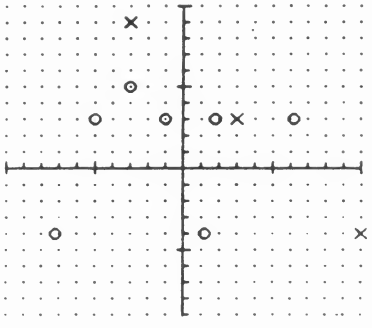
Try y = -4.




It looks like one parabola opens down.

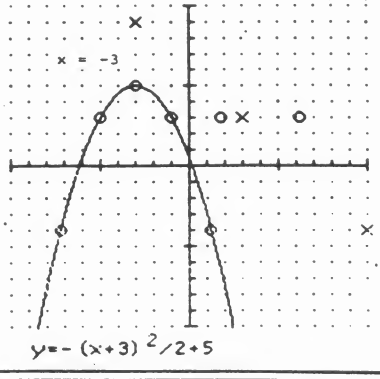
Try to locate the vertex. Use a vertical probe through the parabola, halfway between the sides.

Try x = -3.



The vertex must be at (-3,5). The X marked at (-3,9) marks a second point on the hidden line graph.

Now write an equation for the parabola you found. Send a tracker by entering that equation.




You need to write an equation for the line also. Finally, you will locate and track the other parabola, using methods similar to those above.

SUNBURST COMMUNICATIONS © 1985

TRACKER
#19 - RECORDING SHEET

Practice Section

Check the parts you have completed.

| | one graph | two graphs |
|---|---|---|
| straight line | _____ | _____ |
| parabola | _____ | _____ |
| circle | _____ | _____ |
| ellipse | _____ | _____ |
| hyperbola | _____ | _____ |

---

Novice Game

1. Choice of graph types _____ _____

   Score _____ Number of probes used _____

   Strategies used _____

   _____

2. Choice of graph types _____ _____

   Score _____ Number of probes used _____

   Strategies used _____

   _____

---

Expert Game

1. Number of graphs _____

   List of graphs discovered _____

   Score _____ Number of probes _____ Number of Trackers____

   Strategies used _____

   _____

2. Number of graphs _____

   List of graphs discovered _____

   Score _____ Number of probes _____ Number of Trackers____

   Strategies used _____

   _____

SUNBURST COMMUNICATIONS © 1985

## APPLE: WORKING WITH THE COMPUTER

1. Turn on the television monitor.

2. Insert the diskette into the disk drive with the label facing up and on the right.

3. Close the door to the disk drive.

4. Turn on the Apple. (The on-off switch is on the back left side of the computer.)

5. You will see a red light on the disk drive turn on. If the disk drive does not turn on in about ten seconds, turn the Apple off and make sure your diskette is placed correctly in the disk drive.

6. The SUNBURST logo will appear on the screen.

7. Follow directions given in the program.

8. If at any time during the program you want to exit, press the ESC key.

### Turning Off the Computer

1. Remove the diskette from the disk drive and return it to its place of storage.

2. Turn off the Apple.

3. Turn off the television or monitor.

" WHAT HAPPENS IF...?" -- SUNBURST COURSEWARE AND WARRANTY

1. What happens if a program will not load or run?
   Call us on our toll-free number and we will send you a new tape or diskette.

2. What if I find an error in the program?
   We have thoroughly tested the programs that SUNBURST carries so we hope this does not happen. But if you find an error, please note what you did before the error occurred. Also, if a message appears on the screen, please write the message down. Then fill out the evaluation form and call us with the information. We will correct the error and send you a new diskette.

3. What happens if the courseware is accidentally destroyed?
   SUNBURST has a lifetime guarantee on its courseware. Send us the product that was damaged and we will send you a new one.

4. How do I stop the program in the middle to go on to something new?
   Press the ESCAPE key several times to return to the main menu.

5. Can I copy this diskette?
   The material on the diskette is copyrighted. You <u>should not</u> copy the diskette.

6. Can I take the diskette out of the disk drive and put it into another computer?
   Yes, after you have loaded the program you want. However, the programs "Green Globs" and "Tracker" will not be able to save or read record scores and games unless the diskette is in the disk drive.

# Sunburst Lifetime Warranty Registration Form

Congratulations on your purchase of quality SUNBURST courseware! To register ownership and protect your lifetime warranty, please complete and return this card. The SUNBURST warranty—unlike any other—guarantees replacement of any program component that becomes lost or damaged during normal use. This warranty applies as long as the program is offered for sale.

USER NAME

TITLE

SCHOOL OR BUSINESS NAME

ADDRESS

CITY STATE ZIP

GRADE LEVEL OF SCHOOL

PRODUCT NAME COMPUTER SYSTEM

PRODUCT NO. DATE PURCHASED

MAY WE SEND A CATALOG TO A FRIEND?

NAME

ADDRESS

CITY STATE ZIP

**I learned about SUNBURST products through:**

- ☐ a SUNBURST catalog
- ☐ another catalog
- ☐ sales person
- ☐ a computer store
- ☐ computer conference
- ☐ friend's recommendation
- ☐ magazine advertisement.
  Name of magazine: _____
- ☐ magazine article.
  Name of magazine: _____

The education or computer magazine I read most is: _____

**The three most important factors influencing my purchase decision are:**

- ☐ classroom tested
- ☐ preview policy
- ☐ subject matter
- ☐ reputation of publisher
- ☐ program's use of graphics and sound
- ☐ instructional quality
- ☐ price
- ☐ options to edit
- ☐ guide and support material
- ☐ ease of use
- ☐ other: _____

We hope you enjoy your SUNBURST product!

Please help us serve you better by filling out and returning the enclosed evaluation form. . . . We're waiting to hear from you!

NO POSTAGE
NECESSARY
IF MAILED
IN THE
UNITED STATES

BUSINESS REPLY MAIL

FIRST CLASS PERMIT NO. 53 PLEASANTVILLE, NY

POSTAGE WILL BE PAID BY ADDRESSEE

**SUNBURST COMMUNICATIONS**
39 Washington Avenue
Pleasantville, New York 10570